# Back-to-back dijet production in DIS: interplay of two QCD formalisms beyond leading power approximations

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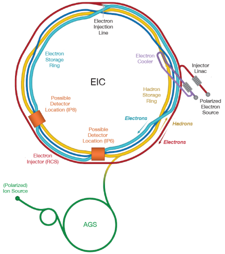
with Tolga Altinoluk, Alina Czajka and Cyrille Marquet, (arXiv:2410.00612 [hep-ph]).

DBP Annual Reporting Seminar
5 grudnia 2024 r.

# The Electron-Ion Collider and the BP2 QCD theory group

A large part of activities of the BP2 QCD theory group are motivated by the future Electron-Ion Collider:

- EIC expected to start in the early 2030s at Brookhaven (USA)
- Colliding electrons with protons or nuclei from deuteron to uranium
- Polarized electron and proton/light ion beams
- Electron-proton c.o.m. energy from $20$ to $140$ GeV
- Electron-nucleon luminosity $10^{33}$ to $10^{34}$ cm$^{-2}$ s$^{-1}$




Main physics program of the EIC:

1. Spatial distribution of partons in proton/nuclei: Exclusive scattering processes
   $\rightarrow$ formalism of Generalized Parton Distributions
2. Momentum distribution of partons in proton/nuclei (+spin): Semi-inclusive scattering processes
   $\rightarrow$ formalism of Transverse Momentum Dependent (TMD) Parton Distributions
3. Non-linear QCD dynamics (Gluon saturation): High-energy/low x scattering processes
   $\rightarrow$ formalisms of Dipole factorization and Color Glass Condensate (CGC)

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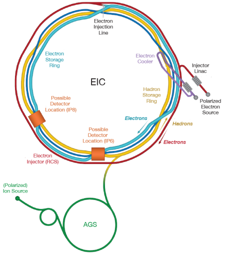


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   Prof. dr hab. Lech Szymanowski, dr hab. Jakub Wagner, dr hab. Pawel Sznajder, dr Minhuan Chu, dr Victor Martinez Fernandez (graduated in oct. 2024)

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3. Non-linear QCD dynamics (Gluon saturation): High-energy/low x scattering processes
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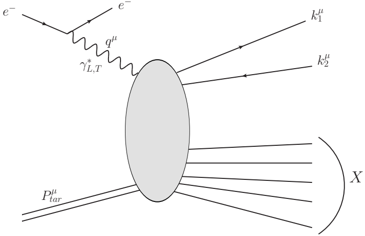
   dr hab. Tolga Altinoluk, dr Guillaume Beuf, dr Alina Czajka, dr Etienne Blanco, dr Pedro Agostini (until sept. 2024), Swaleha Mulani, Jules Favrel, Prof. dr Jamal Jalilian-Marian (visited for 6 months with ULAM grant), dr Josh Tawabutr (joining soon), dr Michael Fucilla (joining soon for 2 years with ULAM grant).

# Dijet production in deep inelastic scattering (DIS)




Photon virtuality: $Q^2 = -q_\mu q^\mu$

Photon-target center of mass energy:
$W^2 = (q + P_{tar})^2$

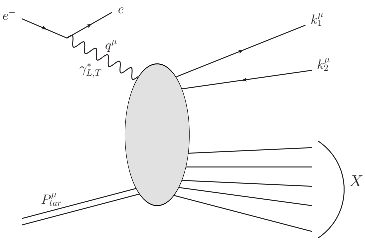
Measured jets: $k_1^\mu$ and $k_2^\mu$

Conventions:

Light-cone variables: $x^\pm = \frac{(x^0 \pm x^3)}{\sqrt{2}}$

Photon going in positive direction along the $x^3$ axis, and proton or nucleus target along the negative direction
$\Rightarrow W^2 \sim 2q^+ P^-_{tar}$ at high collision energy

# TMD factorization for DIS dijet




Change of variables for the jets:

Jets light-cone momentum fractions:
$z_1 = k_1^+/(k_1^+ + k_2^+)$ and
$z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$

Dijet imbalance: $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$

Relative jet momentum:
$\mathbf{P} = (z_2 \mathbf{k}_1 - z_1 \mathbf{k}_2)$

Back-to-back (hard) jets production regime: $|\mathbf{k}| \ll |\mathbf{P}| \sim W$

$\rightarrow$ TMD factorization, valid up to power-suppressed corrections in $|\mathbf{k}|/|\mathbf{P}|$ :

$$\frac{d\sigma_{\gamma^*_{T,L}\rightarrow q_1 \bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|_{\text{TMD}} = \mathcal{H}^{f_1^g}_{T,L}(z_1, \mathbf{P}, Q^2)\, \mathrm{x}\, f_1^g(\mathrm{x}, \mathbf{k}) + \mathcal{H}^{h_1^{\perp g}}_{T,L}(z_1, \mathbf{P}, Q^2)\, \mathrm{x}\, h_1^{\perp g}(\mathrm{x}, \mathbf{k})\bigg|_{\mathrm{x}=\frac{\mathbf{P}^2+m^2}{z_1 z_2 W^2}+\frac{Q^2}{W^2}}$$

- Unpolarized gluon TMD distribution $f_1^g(\mathrm{x}, \mathbf{k})$
- Linearly polarized gluon TMD distribution $h_1^{\perp g}(\mathrm{x}, \mathbf{k})$

$\rightarrow$ Picks a gluon in the target with momentum $(\mathrm{x}P^-_{tar}, \mathbf{k})$

(At higher order in $\alpha_s$, extra dependence on two factorization scales, from CSS resummation of large logs)

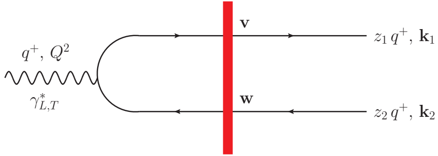
# Dipole factorization for DIS dijet

Other regime: Regge-Gribov high-energy limit: $Q^2$, $\mathbf{k}_1^2$, $\mathbf{k}_2^2 \ll W^2 \to +\infty$

$\to$ Dipole factorization, valid up to power-suppressed corrections in $\mathbf{k}_{1,2}^2/W^2$

Leading power at high energy corresponds to Eikonal approximation:

- Static gluon field $\mathcal{A}^\mu(z) = \mathcal{A}^\mu(z^+, \mathbf{z}, 0)$ due to large Lorentz time dilation of the target
- Shockwave limit $\mathcal{A}^\mu(z) \propto \delta(z^+)$ due to large Lorentz length contraction of the target
- Only $\mathcal{A}^-$ component of gluon field is enhanced by large boost of the target




DIS dijet amplitude in the dipole factorization:

$$i\mathcal{M}^{\text{Eik}}_{q_1 \bar{q}_2 \leftarrow \gamma^*_{L,T}} \sim \int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}\, f_{L,T}(z_1, \mathbf{v}-\mathbf{w}, Q^2) \left[\mathcal{U}_F\Big(\mathbf{v}\Big)\mathcal{U}_F^\dagger\Big(\mathbf{w}\Big) - 1\right]$$

Coherent multiple scattering of quark and antiquark on the target resummed via Wilson line:

$$\mathcal{U}_F(\mathbf{z}) \equiv \mathbf{1} + \sum_{N=1}^{+\infty} \frac{1}{N!}\, \mathcal{P}_+ \left[-ig \int dz^+\, t\cdot\mathcal{A}^-(z^+, \mathbf{z}, 0)\right]^N$$

$\Rightarrow$ Non-linear gluon saturation effects included

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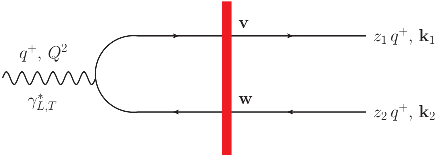


$\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$

$\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

DIS dijet amplitude in the dipole factorization:

$$i\mathcal{M}^{\text{Eik}}_{q_1\bar{q}_2\leftarrow\gamma^*_{L,T}} \sim \int_{\mathbf{r}} e^{-i\mathbf{r}\cdot\mathbf{P}}\, f_{L,T}(z_1,\mathbf{r},Q^2) \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r}\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r}\Big) - 1\right]$$

New variables:

- Dipole vector $\mathbf{r} = \mathbf{v} - \mathbf{w}$, conjugate to $\mathbf{P}$
- Dipole impact parameter $\mathbf{b} = z_1\,\mathbf{v} + z_2\,\mathbf{w}$, conjugate to $\mathbf{k}$

# Interplay between TMD and high-energy approaches

- TMD factorization: leading power (LP) in the back-to-back limit $|\mathbf{k}| \ll |\mathbf{P}| \sim W$
- Dipole factorization: leading power (eikonal) in the high-energy limit $|\mathbf{k}| \sim |\mathbf{P}| \ll W$

Consistency of both approaches known in the double limit $|\mathbf{k}| \ll |\mathbf{P}| \ll \sqrt{s}$ at leading power (Dominguez, Marquet, Xiao, Yuan, 2011)

$\Rightarrow$ How are the two formalisms matching when including subleading power corrections?

- Next-to-leading power corrections (NLP) in the back-to-back jets regime, suppressed as $|\mathbf{k}|/|\mathbf{P}|$
- Next-to-eikonal (NEik) corrections in the high-energy regime, suppressed as $\mathbf{P}^2/W^2$
- Mixed NLP and NEik corrections, suppressed as $|\mathbf{P}||\mathbf{k}|/W^2$

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- Mixed NLP and NEik corrections, suppressed as $|\mathbf{P}||\mathbf{k}|/W^2$

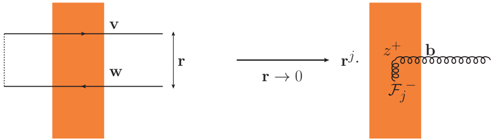
Method:

1. Calculate NEik corrections to the DIS dijet amplitude in the dipole factorization
   - Relaxing the static approximation for the target gluon field
   - Relaxing the shockwave approximation
   - Including the transverse components of the target gluon field
2. Expand the result in the back-to-back jets regime $|\mathbf{k}| \ll |\mathbf{P}|$
3. Interpret the result in terms of TMDs

# Back-to-back regime: small dipole $\mathbf{r}$ expansion

Back-to-back expansion $\mathbf{P} \gg \mathbf{k}$ of the Eikonal DIS dijet amplitude:
$\to$ Equivalent to small dipole $\mathbf{r} \ll \mathbf{b}$ expansion

$$\int_{\mathbf{r}} e^{-i\mathbf{r}\cdot\mathbf{P}}\, f_{L,T}(z_1,\mathbf{r},Q^2) \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[\mathcal{U}_F\Big(\mathbf{b}+z_2\,\mathbf{r}\Big)\mathcal{U}_F^\dagger\Big(\mathbf{b}-z_1\,\mathbf{r}\Big)-1\right]$$
$$= \int_{\mathbf{r}} e^{-i\mathbf{r}\cdot\mathbf{P}}\, f_{L,T}(z_1,\mathbf{r},Q^2) \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \left[z_2\mathbf{r}^j\,(\partial_j\mathcal{U}_F(\mathbf{b}))\mathcal{U}_F^\dagger(\mathbf{b}) - z_1\mathbf{r}^j\mathcal{U}_F(\mathbf{b})(\partial_j\mathcal{U}_F^\dagger(\mathbf{b})) + O(\mathbf{r}^2)\right]$$
$$= \int_{\mathbf{r}} e^{-i\mathbf{r}\cdot\mathbf{P}}\, f_{L,T}(z_1,\mathbf{r},Q^2) \left[\mathbf{r}^j\, t^a \int_{\mathbf{b}} e^{-i\mathbf{b}\cdot\mathbf{k}} \int_{z^+} \mathcal{U}_A\Big(+\infty,z^+;\mathbf{b}\Big)_{ab}(-ig)\mathcal{F}_j^{b\;-}(z^+,\mathbf{b}) + O(\mathbf{r}^2)\right]$$




Note: 0th order in the $\mathbf{r}$ expansion trivial $\to$ first order in $\mathbf{r}$ is the leading power

In our study (Altinoluk, G.B., Czajka, Marquet, arXiv:2410.00612 [hep-ph]):

- Calculation of $O(\mathbf{r}^2)$ terms: NLP in the back-to-back regime
- Small $\mathbf{r}$ expansion also performed for the NEik corrections ($\to$ LP and NLP terms)

# Back-to-back cross section: $\langle \mathcal{F}^{\perp-}\mathcal{F}^{\perp-}\rangle$ contributions

Including all contributions of the form $\langle \mathcal{F}^{\perp-}\mathcal{F}^{\perp-}\rangle$, of order Eik or NEik, and LP or NLP in the back-to-back regime:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{dP.S.}\Bigg|^{\mathcal{F}^{\perp-}\mathcal{F}^{\perp-}} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left[1 + i(z^+ - z'^+)\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+z_1z_2} + \text{NNEik}\right]$$
$$\times\left\langle \mathcal{F}_a^{i-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_b^{j-}(z^+,\mathbf{b})\right\rangle$$

With $\bar{Q}^2 = m^2 + z_1z_2Q^2$

• NEik corrections and NLP corrections in the $\langle \mathcal{F}^{\perp-}\mathcal{F}^{\perp-}\rangle$ contribution factorize from each other

Notation: TMD $\langle \mathcal{F}^{i-}\mathcal{F}^{j-}\rangle$ correlator in unpolarized target (with the target mass $M$):

$$\mathrm{x}\Phi^{i-;j-}(\mathrm{x},\mathbf{k}) = \frac{2}{(2\pi)^3}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+} e^{i\mathrm{x}P^-_{tar}(z^+-z'^+)}\left\langle \mathcal{F}_a^{i-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_b^{j-}(z^+,\mathbf{b})\right\rangle$$
$$= \frac{\delta^{ij}}{2}\,\mathrm{x}f_1^g(\mathrm{x},\mathbf{k}) + \left[\mathbf{k}^i\mathbf{k}^j - \frac{\mathbf{k}^2}{2}\delta^{ij}\right]\frac{1}{2M^2}\,\mathrm{x}h_1^{\perp g}(\mathrm{x},\mathbf{k})$$

- Missing phase in Eikonal contribution $\Rightarrow$ Recovering gluon TMDs but at $\mathrm{x}=0$ from Eikonal result
- NEik correction can be interpreted as start of the expansion of the phase
  $\Rightarrow$ x dependence of TMDs recovered from an all order resummation of power corrections to eikonal approximation
  $\Rightarrow$ DIS dijets production probes TMDs at $\mathrm{x} = \frac{[\mathbf{P}^2+\bar{Q}^2]}{z_1z_2W^2}$: ok!

# Back-to-back cross section: full result

$$\frac{d\sigma_{\gamma^*_{T,L}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|_{\text{Eik+NEik}} = \alpha_{\text{em}} e_f^2\, \alpha_s \bigg\{ \mathcal{C}_{T,L}^{f_1^g}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, f_1^g(\text{x},\mathbf{k}) + \mathcal{C}_{T,L}^{h_1^{\perp g}}(z_1,\mathbf{P},\mathbf{k})\, \text{x}\, h_1^{\perp g}(\text{x},\mathbf{k})$$

$$+ \frac{(\mathbf{k}\cdot\mathbf{P})}{W^2}\, \mathcal{C}_{T,L}^{f^{\perp g}}(z_1,\mathbf{P})\, \text{x}\, f^{\perp g}(\text{x},\mathbf{k}) + \frac{(\mathbf{k}\cdot\mathbf{P})}{W^2}\, \mathcal{C}_{T,L}^{\bar{g}^{\perp g}}(z_1,\mathbf{P})\, \text{x}\, \bar{g}^{\perp g}(\text{x},\mathbf{k}) \bigg\}\bigg|_{\text{x}=\frac{[\mathbf{P}^2+\bar{Q}^2]}{z_1 z_2\, W^2}}$$

$$+ 3\,\mathcal{F}\,\text{terms} + O\left(\frac{1}{\mathbf{P}^6}\right)$$

Extra terms: all power suppressed in the back-to-back jets regime (NLP)

- NLP gluon TMD $f^{\perp g}(\text{x},\mathbf{k})$, defined from $\langle \mathcal{F}^{+-}\mathcal{F}^{\perp -}\rangle$

  $\rightarrow$ NEik correction beyond the static approximation for the target fields

- NLP gluon TMD $\bar{g}^{\perp g}(\text{x},\mathbf{k})$, defined from $\langle \mathcal{F}^{12}\mathcal{F}^{\perp -}\rangle$

  $\rightarrow$ NEik correction due to the transverse components of the target field

- Extra NLP corrections beyond TMD parton distributions, of the type $\langle \mathcal{F}^{\perp -}\mathcal{F}^{\perp -}\mathcal{F}^{\perp -}\rangle$

  $\rightarrow$ Contain both Eik and NEik contributions. Genuine saturation corrections: nonlinear QCD.

# Summary

To understand the interplay between dipole and TMD formalisms, we studied the NEik DIS dijet cross-section in the back-to-back jets limit, including NLP corrections. Various types of contributions are obtained:

- $\langle \mathcal{F}^{i-} \mathcal{F}^{j-} \rangle$: unpolarized and linearly polarized gluon TMDs distributions $f_1^g$ and $h_1^{\perp g}$
  - In the Eikonal approximation: TMDs at $\mathrm{x} = 0$ momentum fraction
  - NEik correction is the first order correction in the Taylor expansion of the TMDs around $\mathrm{x} = 0$
    $\Rightarrow$ $\mathrm{x}$ dependence of the TMDs recovered by resumming terms of all powers beyond the eikonal approximation

- Subleading gluon TMDs contribute to the cross section at NEik and NLP order:
  - $f^{\perp g}$, of the type $\langle \mathcal{F}^{i-} \mathcal{F}^{+-} \rangle$
  - $\bar{g}^{\perp g}$, of the type $\langle \mathcal{F}^{l-} \mathcal{F}^{ij} \rangle$ (for $\gamma_T^*$ case)

- 3 fields correlators $\langle \mathcal{F}^{i-} \mathcal{F}^{j-} \mathcal{F}^{l-} \rangle$ at NLP: beyond TMD partonic distributions

# Final result: $\langle \mathcal{F}\mathcal{F} \rangle$ contributions to the cross sections

$$\frac{d\sigma_{\gamma^*_{T,L}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|_{\text{Eik+NEik}} = \alpha_{\text{em}} e_f^2\, \alpha_s \bigg\{ \mathcal{C}^{f_1^g}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \mathrm{x}\, f_1^g(\mathrm{x},\mathbf{k}) + \mathcal{C}^{h_1^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\, \mathrm{x}\, h_1^{\perp g}(\mathrm{x},\mathbf{k})$$

$$+ \frac{(\mathbf{k}\cdot\mathbf{P})}{W^2}\, \mathcal{C}^{f^{\perp g}}_{T,L}(z_1,\mathbf{P})\, \mathrm{x}\, f^{\perp g}(\mathrm{x},\mathbf{k}) + \frac{(\mathbf{k}\cdot\mathbf{P})}{W^2}\, \mathcal{C}^{\bar{g}^{\perp g}}_{T,L}(z_1,\mathbf{P})\, \mathrm{x}\, \bar{g}^{\perp g}(\mathrm{x},\mathbf{k}) \bigg\}\bigg|_{\mathrm{x}=\frac{[\mathbf{P}^2+\bar{Q}^2]}{z_1 z_2 W^2}}$$

$$+ 3\,\mathcal{F}\ \text{terms} + O\left(\frac{1}{\mathbf{P}^6}\right)$$

Contribution from longitudinal photon exchange:

$$\mathcal{C}^{f_1^g}_{L}(z_1,\mathbf{P},\mathbf{k}) = \frac{8Q^2\, z_1^2 z_2^2}{[\mathbf{P}^2+\bar{Q}^2]^4} \left\{ \mathbf{P}^2 + (z_2-z_1)(\mathbf{k}\cdot\mathbf{P}) \left[ -1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]} \right] \right\} + O\left(\frac{Q^2\mathbf{k}^2}{\mathbf{P}^8}\right)$$

$$\mathcal{C}^{h_1^{\perp g}}_{L}(z_1,\mathbf{P},\mathbf{k}) = \frac{4Q^2\, z_1^2 z_2^2}{[\mathbf{P}^2+\bar{Q}^2]^4} \frac{\mathbf{k}^2}{M^2} \bigg\{ \left( \frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2} - 1 \right) \mathbf{P}^2$$

$$+ (z_2-z_1)(\mathbf{k}\cdot\mathbf{P}) \left[ -1 + \frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]} \left( \frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2} - 1 \right) \right] \bigg\} + O\left(\frac{Q^2\mathbf{k}^2}{\mathbf{P}^8}\right)$$

$$\mathcal{C}^{f_1^{\perp g}}_{L}(z_1,\mathbf{P}) = -\,32Q^2\, z_1 z_2 \frac{(z_2-z_1)[\mathbf{P}^2+m^2]}{[\mathbf{P}^2+\bar{Q}^2]^4} + O\left(\frac{Q^2|\mathbf{k}|}{|\mathbf{P}|^7}\right)$$

$$\mathcal{C}^{\bar{g}^{\perp g}}_{L}(z_1,\mathbf{P}) = 0$$

$$\bar{Q}^2 = m^2 + z_1 z_2 Q^2\,.$$

# Final result: $\langle \mathcal{F}\mathcal{F} \rangle$ contributions to the cross sections

$$\frac{d\sigma_{\gamma^*_{T,L}\to q_1\bar{q}_2}}{dz_1\, d^2\mathbf{P}\, d^2\mathbf{k}}\bigg|_{\text{Eik+NEik}} = \alpha_{\text{em}} e_f^2\, \alpha_s \bigg\{ \mathcal{C}^{f_1^g}_{T,L}(z_1,\mathbf{P},\mathbf{k})\,\mathrm{x}\, f_1^g(\mathrm{x},\mathbf{k}) + \mathcal{C}^{h_1^{\perp g}}_{T,L}(z_1,\mathbf{P},\mathbf{k})\,\mathrm{x}\, h_1^{\perp g}(\mathrm{x},\mathbf{k})$$
$$+ \frac{(\mathbf{k}\cdot\mathbf{P})}{W^2}\,\mathcal{C}^{f^{\perp g}}_{T,L}(z_1,\mathbf{P})\,\mathrm{x}\, f^{\perp g}(\mathrm{x},\mathbf{k}) + \frac{(\mathbf{k}\cdot\mathbf{P})}{W^2}\,\mathcal{C}^{\bar{g}^{\perp g}}_{T,L}(z_1,\mathbf{P})\,\mathrm{x}\,\bar{g}^{\perp g}(\mathrm{x},\mathbf{k}) \bigg\}\bigg|_{\mathrm{x}=\frac{[\mathbf{P}^2+\bar{Q}^2]}{z_1 z_2 W^2}}$$
$$+ 3\,\mathcal{F}\text{ terms} + O\left(\frac{1}{\mathbf{P}^6}\right)$$

Contribution from transverse photon exchange:

$$\mathcal{C}^{f_1^g}_{T}(z_1,\mathbf{P},\mathbf{k}) = -\frac{2\big[(z_1^2+z_2^2)\bar{Q}^2-m^2\big]}{[\mathbf{P}^2+\bar{Q}^2]^4}\left\{\mathbf{P}^2 + (z_2-z_1)(\mathbf{k}\cdot\mathbf{P})\left[-1+\frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]}\right]\right\}$$
$$+ \frac{(z_1^2+z_2^2)}{[\mathbf{P}^2+\bar{Q}^2]^2}\left[1+\frac{2(z_2-z_1)(\mathbf{k}\cdot\mathbf{P})}{[\mathbf{P}^2+\bar{Q}^2]}\right] + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^6}\right)$$
$$\mathcal{C}^{h_1^{\perp g}}_{T}(z_1,\mathbf{P},\mathbf{k}) = -\frac{\big[(z_1^2+z_2^2)\bar{Q}^2-m^2\big]}{[\mathbf{P}^2+\bar{Q}^2]^4}\,\frac{\mathbf{k}^2}{M^2}\left\{\left(\frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2}-1\right)\mathbf{P}^2\right.$$
$$\left. + (z_2-z_1)(\mathbf{k}\cdot\mathbf{P})\left[-1+\frac{4\mathbf{P}^2}{[\mathbf{P}^2+\bar{Q}^2]}\left(\frac{2(\mathbf{k}\cdot\mathbf{P})^2}{\mathbf{k}^2\mathbf{P}^2}-1\right)\right]\right\} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^6}\right)$$
$$\mathcal{C}^{f_1^{\perp g}}_{T}(z_1,\mathbf{P}) = \frac{4(z_2-z_1)}{[\mathbf{P}^2+\bar{Q}^2]^4}\left\{\big[\mathbf{P}^2+\bar{Q}^2+(z_1^2+z_2^2)Q^2\big]\big[\mathbf{P}^2-\bar{Q}^2\big] + 2m^2Q^2\right\} + O\left(\frac{|\mathbf{k}|}{|\mathbf{P}|^5}\right)$$
$$\mathcal{C}^{\bar{g}^{\perp g}}_{T}(z_1,\mathbf{P}) = \frac{4(z_2-z_1)}{[\mathbf{P}^2+\bar{Q}^2]^2} + O\left(\frac{|\mathbf{k}|}{|\mathbf{P}|^5}\right)$$

$$\bar{Q}^2 = m^2 + z_1 z_2 Q^2\,.$$