Cutkosky Cutting Rules in the Deformed Context [1]

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[1] **A. B.**, *Cutkosky rules and 1-loop κ-deformed amplitudes*, Phys. Rev. D Vol. 110 No 10 (2024) 106003

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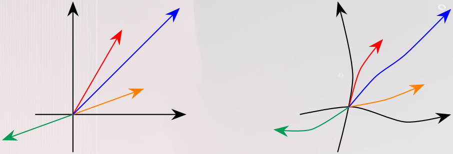
- A popular effective approach is based on non-commutative spacetimes, where particle motion is effectively deformed (with deformation parameter $1/\kappa$).



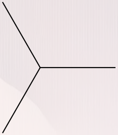
$$\text{sum}((p_1, p_2), (q_1, q_2)) = (p_1 + q_1, p_2 + q_2)$$

$$\text{sum}((p_1^\kappa, p_2^\kappa), (q_1^\kappa, q_2^\kappa)) = (p_1^\kappa \oplus q_1^\kappa, p_2^\kappa \oplus q_2^\kappa)$$

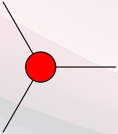
$$\text{negative}((p_1, p_2)) = (-p_1, -p_2)$$

$$\text{negative}((p_1^\kappa, p_2^\kappa)) = (S(p_1^\kappa), S(p_2^\kappa))$$

Non-trivial composition of momenta $\Longrightarrow$ non-trivial rules for conservation of momenta



$$\longrightarrow \qquad \delta(p_1 + p_2 + p_3)$$



$$\longrightarrow \qquad \begin{array}{c} \delta(p_1 \oplus p_2 \oplus p_3) \\ \delta(p_2 \oplus p_1 \oplus p_3) \\ \dots \end{array}$$

Even free particles are non-trivial

$$p = -(-p) \qquad p = S(S(p)) \qquad p \neq -S(p)$$

Greenberg's theorem relates CPT invariance with Lorentz invariance of a theory. According to the theorem,

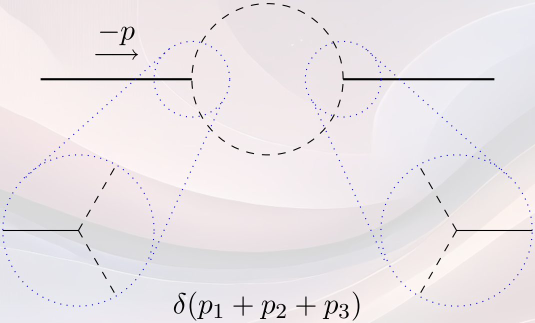
$$\boxed{\text{Lorentz invariance } \Leftrightarrow CPT \text{ invariance}}$$

In the deformed context, in general one can have a CPT and (deformed) Lorentz invariant action, and yet particles and antiparticles can have different decay times.

Greenberg's theorem does not work: $p \neq -S(p)$.

Consider now an unstable particle in the non-deformed context. What is the particle life time?

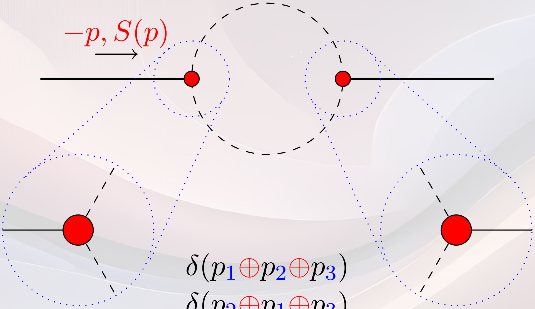
$\Pi =$  $\Longrightarrow \Im(\Pi) \Longrightarrow \tau$



Because of CPT, particles and antiparticles have the same lifetime.

Consider now an unstable particle in the deformed context. What is the particle life time?

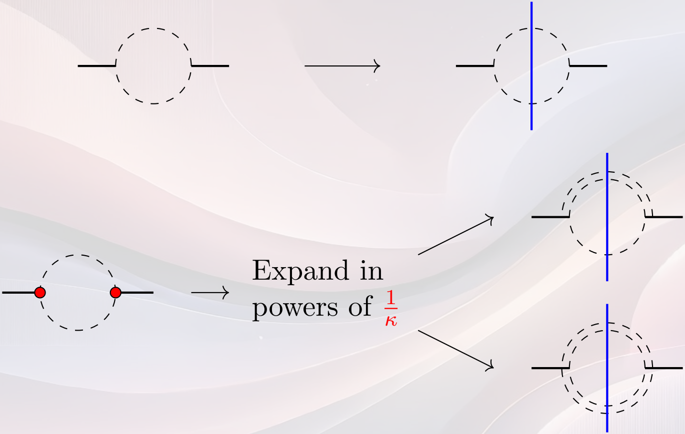
$\Pi =$  $\Longrightarrow \Im(\Pi^{\kappa}) \Longrightarrow \tau^{\kappa}$



$$\delta(p_1 \oplus p_2 \oplus p_3)$$
$$\delta(p_2 \oplus p_1 \oplus p_3)$$
$$\dots$$

Particles and antiparticles do not behave necessarily in the same way!

Despite the differences, it turns out that the same tool can extract $\Im(\Pi)$ both in the (general) deformed and the non-deformed case: Cutkosky cutting rules.



- **Theory:** In the non-deformed SM, Cutkosky rules are related to the unitarity of the scattering matrix S. In the deformed case, establishing the Cutkosky rules for general amplitudes may have very important consequences!
  1 deformed loop ✓

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- **Phenomenology:** We can measure particle/antiparticle decays. Thanks to the validity of the Cutkosky cutting rules, we can exactly and straightforwardly compute the decay width for a wide range of deformed models, and test their predictions against data.

Prospects:

- Often general amplitudes are not well defined.

$$\delta(k \oplus p_1 \oplus p_2), \quad \delta(p_1 \oplus p_2 \oplus k), \quad \delta(p_1 \oplus k \oplus p_2)$$

$$|p_1\rangle|p_2\rangle \neq |p_2\rangle|p_1\rangle$$

Many interesting challenges ahead!

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  Many interesting challenges ahead!

- Find out which deformed models may be excluded by current data, or which are more strictly constrained, and understand why.

Thank you