A new generic and structurally stable cosmological model without singularity

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A R T I C L E I N F O

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A B S T R A C T

Dynamical systems methods are used to investigate a cosmological model with non-minimally coupled scalar field and asymptotically quadratic potential function. We found that for values of the non-minimal coupling constant parameter $\frac{3}{16} < \xi < \frac{1}{4}$ there exists an unstable asymptotic de Sitter state, free from a parallelly propagated singularity for $\frac{5}{24} \leq \xi < \frac{1}{4}$, giving rise to non-singular beginning of the universe. The energy density associated with this state depends on value of the non-minimal coupling constant and can be much smaller than the Planck energy density. For $\xi = \frac{1}{4}$ we found that the initial state is in form of the static Einstein universe. Proposed evolutional model, contrary to the seminal Starobinsky's model, do not depend on the specific choice of initial conditions in phase space, moreover, a small change in the model parameters do not change the evolution thus the model is generic and structurally stable. The values of the non-minimal coupling constant can indicate for a new fundamental symmetry in the gravitational theory. We show that Jordan frame and Einstein frame formulation of the theory are physically nonequivalent.

General Theory of Relativity

$$S = \frac{1}{2\kappa^2} \int \mathrm{d}^4 x \sqrt{-g} \left(R - 2\Lambda \right) + S_m$$

Friedmann-Robertson-Walker symmetry

$$\left(\frac{H}{H_0} \right)^2 = \Omega_{\Lambda,0} + \Omega_{m,0} \left(\frac{a}{a_0} \right)^{-3} + \Omega_{r,0} \left(\frac{a}{a_0} \right)^{-4} + \dots$$

$$S_T = S - \frac{1}{2} \int \mathrm{d}^4 x \sqrt{-g} \left(\nabla^\alpha \phi \, \nabla_\alpha \phi + 2U(\phi) \right)$$

- inflationary epoch – inflaton
- current accelerated expansion – quintessence

The theory

We start from the total action of the theory

$$S = S_g + S_\phi,$$

consisting of the gravitational part described by the standard Einstein-Hilbert action integral

$$S_g = \frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\,R\,,$$

where $\kappa^2 = 8\pi G$, and the matter part of the theory is in the form of non-minimally coupled scalar field

$$S_\phi = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\Big(\varepsilon\nabla^\alpha\phi\,\nabla_\alpha\phi + \varepsilon\xi R\phi^2 + 2U(\phi)\Big)\,,$$

where $\varepsilon = +1, -1$ corresponds to the canonical and the phantom scalar field, respectively.

An asymptotically quadratic potential function

Inflationary paradigm:
"plateau-like" potential functions $\frac{U'(\phi)}{U(\phi)} \to 0$ as $\phi \to \infty$

Working assumptions:
Let us assume, that starting from some values of the scalar field $\phi > \phi^*$ the potential function can be approximated as

$$U(\phi) = \pm\frac{1}{2}m^2\phi^2 \pm M^{4+n}\phi^{-n}\,,$$

where $n > -2$ and the second term constitutes small, asymptotically vanishing deviation.

Projective coordinates and analysis at infinity

We introduce the following dimensionless phase space variables

$$u = \frac{\dot{\phi}}{H\phi}\,, \quad v = \frac{\sqrt{6}}{\kappa}\frac{1}{\phi}\,,$$

and dimensionless parameters describing potential function of the scalar field

$$\mu = \pm\frac{m^2}{H_0^2}\,, \quad \alpha = \pm 2\frac{M^{4+n}}{H_0^2}\left(\frac{\kappa}{\sqrt{6}}\right)^{2+n}\,,$$

where H_0 is the present value of the Hubble function.

Instability of the initial de Sitter state

We investigate dynamical behaviour in the vicinity of the critical point

$$u^{*} = -\frac{2\xi}{1-4\xi}\,, \quad v^{*} = 0\,.$$

The energy conservation condition calculated at this point gives

$$\frac{H^2}{H_0^2}\bigg|^{*} = -\varepsilon\mu\frac{(1-4\xi)^2}{2\xi(1-6\xi)(3-16\xi)} > 0\,,$$

and this quantity must be positive in order to obtain the asymptotic state in the physical region of the phase space.
– an unstable node for

$$\varepsilon\mu < 0 \,\wedge\, \frac{3}{16} < \xi < \frac{1}{4}\,.$$

Non-minimal coupling and $U(\phi) = \pm\frac{1}{2}m^2\phi^2 \pm M^{4+n}\phi^{-n}$

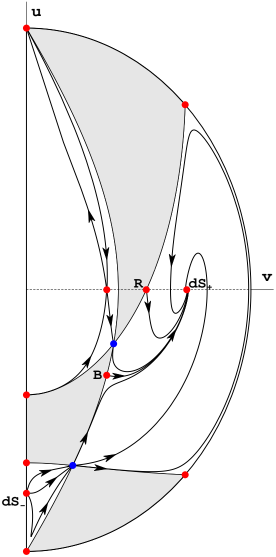


Figure: $\varepsilon = +1$, $\xi = \frac{7}{32}$, $n = 1$, $\mu = -1$, $\alpha = 3$

The special case $\xi = \frac{1}{4}$

New projective coordinate

$$\hat{u} = \frac{1}{u} = H\frac{\phi}{\dot{\phi}}$$

We are interested in dynamics in vicinity of the critical point

$$\hat{u}^* = 0\,, \quad v^* = 0$$

Simple inspection gives that at the asymptotic state the energy conservation condition vanishes together with the cosmological time derivative of the Hubble function

$$\frac{H^2}{H_0^2}\bigg|^* = 0\,, \quad \frac{\dot{H}}{H_0^2}\bigg|^* = 0\,,$$

which gives rise to the Einstein static universe. The acceleration equation calculated at this state is

$$\frac{\dot{H}}{H^2}\bigg|^* = -3\,,$$

which suggest that the Einstein static state under considerations is filled with effective substance in the form of Zeldovich stiff matter with equation of state parameter $w_{\text{eff}} = 1$.

$\xi = \frac{1}{4}$

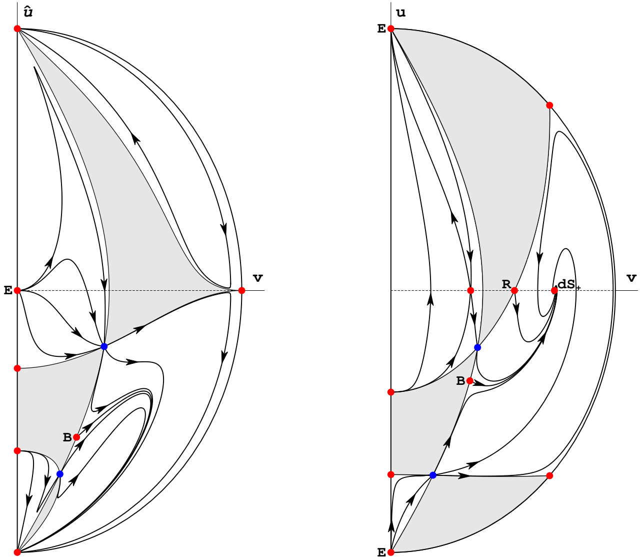


Figure: $\varepsilon = +1$, $\xi = \frac{1}{4}$, $n = 1$, $\mu = -1$, $\alpha = 3$

Physical nonequivalence of Jordan and Einstein frame

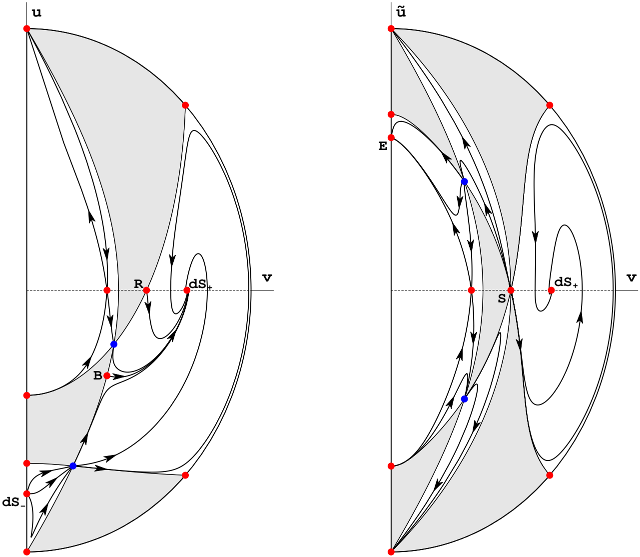


Figure: $\varepsilon = +1$, $\xi = \frac{7}{32}$, $n = 1$, $\mu = -1$, $\alpha = 3$

Non-minimal coupling and the conformal invariance

The Klein-Gordon equation for the scalar field with a monomial potential function is in the following form

$$\Box\phi - \xi R\phi - \varepsilon\, n\, U_0 \phi^{n-1} = 0\,.$$

Using appropriate conformal or Weyl transformation we find

$$\begin{aligned} \tilde{\Box}\tilde{\phi} - \xi\tilde{R}\tilde{\phi} - \varepsilon\, n\, U_0\tilde{\phi}^{n-1} &= \\ \Omega^{-\frac{D+2}{2}}\left(\Box\phi - \xi R\phi - \varepsilon\, n\, U_0\phi^{n-1}\right) &= 0\,, \end{aligned}$$

and this equation holds iff the parameters are

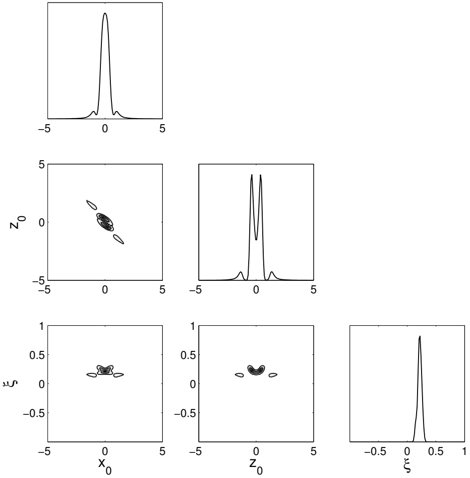
$$\xi = \xi_{\mathrm{conf}} = \frac{1}{4}\frac{D-2}{D-1}\,, \quad n = n_{\mathrm{conf}} = \frac{2D}{D-2}\,.$$

Non-minimal coupling and the conformal invariance

Hence, we obtain the following discrete set of theoretically motivated values of the non-minimal coupling constant and the exponent of a monomial scalar field potential function suggested by the conformal invariance condition in $D \geq 2$ space-time dimensions

$$\Big\{(D, \xi, n)\Big\} = \Big\{ (2, 0, \infty), \left(3, \frac{1}{8}, 6\right), \left(4, \frac{1}{6}, 4\right), \left(5, \frac{3}{16}, \frac{10}{3}\right), \ldots, \left(\infty, \frac{1}{4}, 2\right) \Big\}.$$

Observational constraints



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Observational data:
Union2.1+H(z)+Alcock-Paczyński test

Conclusions

- We have found that for generic scalar field potential functions which asymptotically tend to a quadratic form at infinite values of the scalar field there is an unstable critical point corresponding to: the de Sitter state for non-minimal coupling constant $\frac{3}{16} < \xi < \frac{1}{4}$; and static Einstein universe filled with effective matter in the form of Zeldovich stiff matter for the non-minimal coupling constant $\xi = \frac{1}{4}$.
- Using dynamical systems methods we were able to directly compare dynamics of the model in original Jordan frame and conformally transformed Einstein frame. We have shown that both descriptions are physically nonequivalent since the initial unstable de Sitter state in the Jordan frame is transformed in to the stable Einstein static state in the Einstein frame.

Rather than conclusions

Is conformal invariance the fundamental symmetry ?