Entanglement and symmetry in high-energy scattering

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in collaboration with

Enrico Maria Sessolo

based on:
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Annual Department Seminar

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National Science Centre Poland

SONATA BIS 7 (PI: K. Kowalska)
SONATA BIS 10 (PI: E.M.Sessolo)

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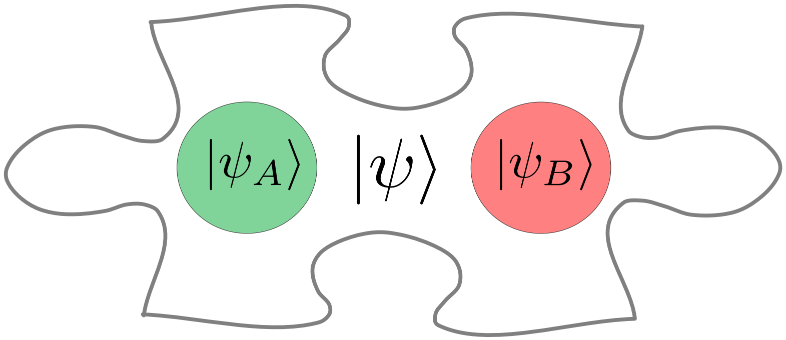
Entanglement bit by bit



$$|\psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

entanglement = non-separability

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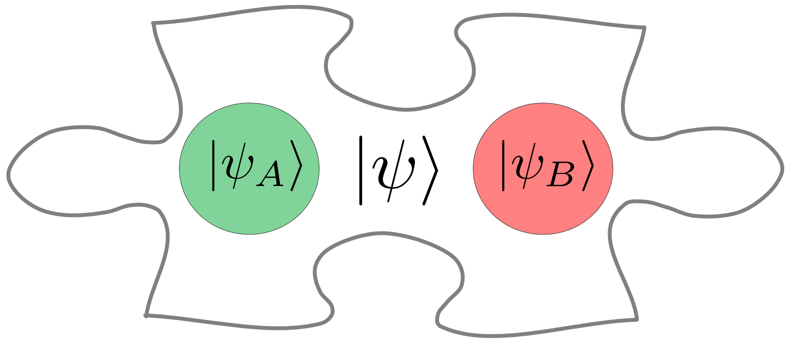

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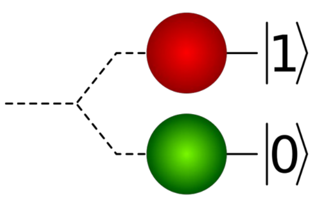
entanglement = non-separability

Example: **a qubit**

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



$|\psi\rangle \in \mathbb{C}^2 \quad |\alpha|^2 + |\beta|^2 = 1$

two-qubit state

$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$|\psi\rangle \in \mathbb{C}^2 \times \mathbb{C}^2 \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$

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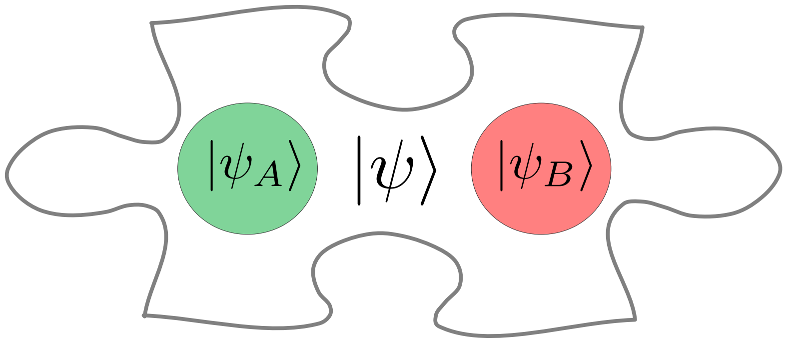

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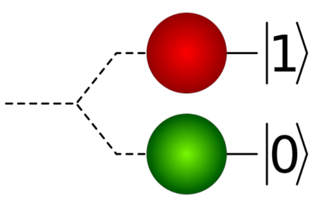

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1. separable (non-entangled)

$$|\psi\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle) \otimes (\alpha_2|0\rangle + \beta_2|1\rangle) =$$

$$\alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$$

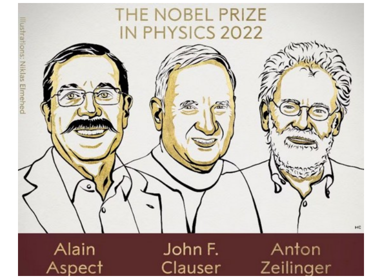
2. non-separable (entangled)

$$|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle \pm \frac{1}{\sqrt{2}}|11\rangle \qquad |\psi\rangle = \frac{1}{\sqrt{2}}|01\rangle \pm \frac{1}{\sqrt{2}}|10\rangle$$

Entanglement is hot!

Measured experimentally (Bell inequalities violation)
for linear polarization of low-energy photons

Clauser et al., ‘72; Aspect et al., ‘82; Zeilinger et al., ‘98;



… also at colliders
for spin of the top quarks

ATLAS ‘23; CMS ‘24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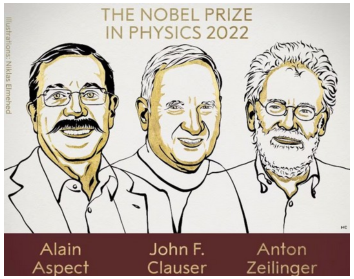

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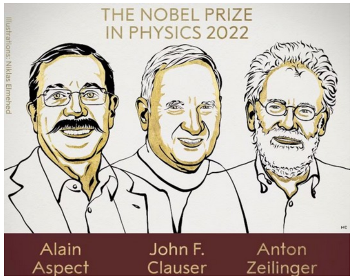

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Applications in particle physics

- **quantum tomography** ex. Afik, de Nova, EPJ Pus 136 (9) (2021) 907

  use entanglement (and other quantum observables) to enhance sensitivity of collider searches

- **theoretical implications** ⇒ **emergent symmetries**

  investigate connections between entanglement and fundamental properties of quantum field theories

Emergent symmetries

Symmetries arising from **entanglement extremization**
in scattering processes

- non-relativistic baryon-baryon scattering → SU(4) and SU(16) global symmetries from **minimizing** entanglement among spins

  Beane, Kaplan, Klco, Savage,
  PRL 122 (2019) 102001

- quantum electrodynamics → U(1) gauge symmetry from **maximizing** entanglement among helicities

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- Two-Higgs-doublet model → indications of SO(8) global symmetry from **minimizing** entanglement among Higgs boson “flavors”

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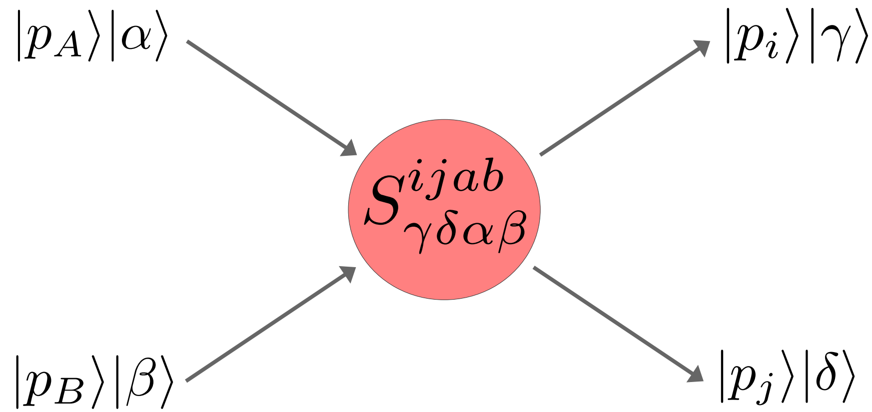
→ Red flags
- one (out of several) scattering channel was considered
- their formalism does not preserve unitarity – can't quantify entanglement exactly

Entanglement in scattering

$$|\mathrm{in}\rangle = \frac{1}{\sqrt{V}}\,|\mathbf{p}_A\mathbf{p}_B\rangle \otimes \sum_{\alpha,\beta=1,2} a_{\alpha\beta}|\alpha\beta\rangle$$



$$|\mathrm{out}\rangle = S\,|\mathrm{in}\rangle$$

Hilbert space: **momentum** + **flavor** (qubit)

$$\mathcal{H}_{\mathrm{tot}} = L^2(\mathbb{R}^3 \otimes \mathbb{R}^3) \otimes \mathbb{C}^4$$

<u>In perturbation theory:</u>

$$S^{ijab}_{\gamma\delta\alpha\beta} = (\mathcal{I} + iT)^{ijab}_{\gamma\delta\alpha\beta}$$

$$= (2\pi)^6\, 4\, E_a E_b\, \delta^{ijab}_{\gamma\delta\alpha\beta} + (2\pi)^4 \delta^4(p_a + p_b - p_i - p_j)\, i\mathcal{M}_{\gamma\delta,\alpha\beta}(p_a, p_b \to p_i, p_j)$$

The final-state density matrix:

$$\rho = |\mathrm{out}\rangle\langle\mathrm{out}|$$

encodes all the properties of a quantum system **(entanglement)**

Perturbative density matrix

$$\rho = |\mathrm{out}\rangle\langle\mathrm{out}|$$

Shao et al., ‘08; Seki et al., ‘15; Peschanski, Seki, ‘16, ‘19; Carney et al., ‘16; Rätzel et al., ‘17; Fan et al., ‘17; Araujo et al., ‘19; Fan et al., ‘21; Fonseca et al., ‘22; Shivashankara ‘23; Aoude et al., ‘24

$$\mathrm{Tr}(\rho) = 1$$ ⟹ unitarity of the *S*-matrix

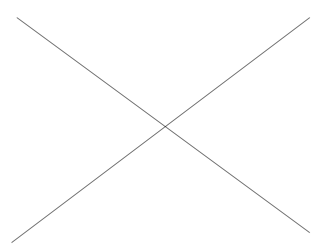
optical theorem

$$\langle\mathrm{out}|\mathrm{out}\rangle = 1 + \Delta\left(i \sum_{\alpha\beta,\gamma\delta} a^*_{\alpha\beta}\mathcal{M}_{\alpha\beta,\gamma\delta}(p_A,p_B \to p_A,p_B)a_{\gamma\delta} + \mathrm{c.c.}\right)$$
$$+\ \Delta \int\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\frac{d^3p_j}{(2\pi)^3}\frac{1}{2E_j}(2\pi)^4\delta^4(p_A+p_B-p_i-p_j)$$
$$\times \sum_{\alpha\beta,\rho\epsilon,\sigma\tau} \mathcal{M}_{\alpha\beta,\rho\epsilon}(p_A,p_B\to p_i,p_j)a_{\rho\epsilon}\mathcal{M}^*_{\alpha\beta,\sigma\tau}(p_A,p_B\to p_i,p_j)a^*_{\sigma\tau}$$

$$\Delta = \frac{(2\pi)^4\delta^4(p_A+p_B-p_A-p_B)}{4E_AE_B\left[(2\pi)^3\,\delta^3(0)\right]^2}$$

(indeterminate normalization)

We need to work at 1-loop order Carena et al. ‘24 only tree level



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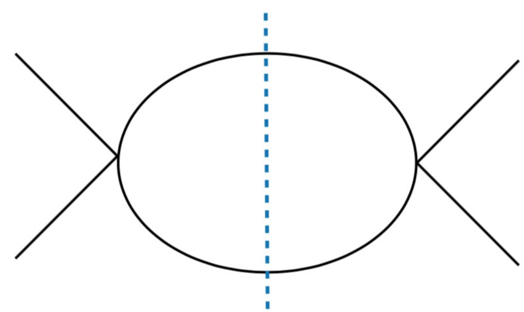
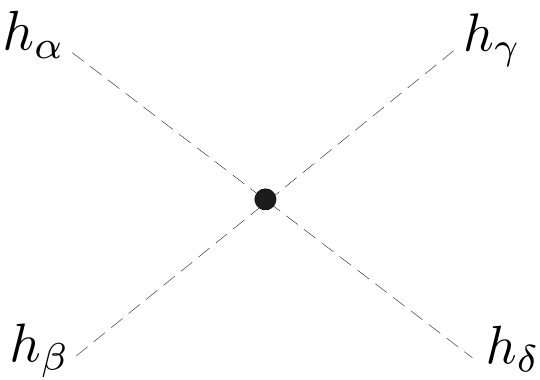


2HDM in a nutshell

inert SU(2) doublets: $H_\alpha = \begin{pmatrix} h^+_\alpha \\ h^0_\alpha \end{pmatrix}_{Y=\frac{1}{2}}$ $\quad \alpha = 1, 2 \to |1\rangle, |2\rangle$ **two flavors**

scalar potential:
$$V(H_1, H_2) = \mu_1^2\, H_1^\dagger H_1 + \mu_2^2\, H_2^\dagger H_2 + \left(\mu_3^2\, H_1^\dagger H_2 + \text{H.c.}\right)$$
$$+ \lambda_1\, (H_1^\dagger H_1)^2 + \lambda_2\, (H_2^\dagger H_2)^2 + \lambda_3\, (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4\, (H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \left(\lambda_5\, (H_1^\dagger H_2)^2 + \lambda_6\, (H_1^\dagger H_1)(H_1^\dagger H_2) + \lambda_7\, (H_2^\dagger H_2)(H_1^\dagger H_2) + \text{H.c.}\right)$$

contact interactions



$$i\,\mathcal{M}^{(0+1)}_{\gamma\delta,\alpha\beta}$$

only this channel considered in Carena et al. '24

$$i\mathcal{M}^{(0)}(h^+h^0 \to h^+h^0) = -i \begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & 2\lambda_5 \\ \lambda_6 & \lambda_3 & \lambda_4 & \lambda_7 \\ \lambda_6 & \lambda_4 & \lambda_3 & \lambda_7 \\ 2\lambda_5 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^+h^{0*} \to h^+h^{0*}) = -i \begin{pmatrix} 2\lambda_1 & \lambda_6 & \lambda_6 & \lambda_4 \\ \lambda_6 & \lambda_3 & 2\lambda_5 & \lambda_7 \\ \lambda_6 & 2\lambda_5 & \lambda_3 & \lambda_7 \\ \lambda_4 & \lambda_7 & \lambda_7 & 2\lambda_2 \end{pmatrix}$$

$$i\mathcal{M}^{(0)}(h^0h^0 \to h^0h^0) = -i \begin{pmatrix} 4\lambda_1 & 2\lambda_6 & 2\lambda_6 & 4\lambda_5 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 2\lambda_6 & \lambda_3+\lambda_4 & \lambda_3+\lambda_4 & 2\lambda_7 \\ 4\lambda_5 & 2\lambda_7 & 2\lambda_7 & 4\lambda_2 \end{pmatrix}$$

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Q: any constraints on λ from entanglement extremization?

Entanglement creation

no initial entanglement: $|\mathrm{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle |11\rangle$

$|\mathrm{out}\rangle = S\,|\mathrm{in}\rangle$ $\qquad \rho^F = \mathrm{Tr}_p(\rho)$ **reduced density matrix** (tracing out the momentum)

post-scattering entanglement:

von Neuman entropy

eigenvalues of ρ^F

$$S_N(\rho^F) = -\sum_i \theta_i \log_2 \theta_i$$

entanglement between **flavor and momentum**

with

$$\theta_1 = 1 - \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) + 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$$\theta_2 = \Delta\left(\frac{\lambda_5^2}{\pi} + \frac{\lambda_6^2}{2\pi}\right) - 16\Delta^2\left(\lambda_5^2 + \frac{\lambda_6^2}{2}\right)$$

$h^0 h^0 \to h^0 h^0$

concurrence

$$C(\rho^F) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$$

λ: eigenvalues of $\rho^F(\sigma_y \otimes \sigma_y)\rho^{F*}(\sigma_y \otimes \sigma_y)$

entanglement between **flavor qubits**

$$C(\rho^F) = \sqrt{\frac{2\Delta\lambda_5^2}{\pi} + 32\Delta^2\lambda_5^2}$$

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Repeating for |12>, |21>, |22> (all channels):

couplings that generate entanglement

$\lvert\text{in}\rangle_F$	momentum-flavor space	two-flavor space
$\lvert 11\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6$	$\lambda_3,\ \lambda_4,\ \lambda_5$
$\lvert 12\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$
$\lvert 21\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_6,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$
$\lvert 22\rangle$	$\lambda_3,\ \lambda_4,\ \lambda_5,\ \lambda_7$	$\lambda_3,\ \lambda_4,\ \lambda_5$

$\lambda_3 = \lambda_4 = \lambda_5 = 0$ different from $\lambda_1 = \lambda_2 = \lambda_3 \neq 0$

Carena et al. '24

⇒ **No symmetry from entanglement minimization**

Entanglement transformation

maximal initial flavor entanglement: $|\text{in}\rangle = \frac{1}{\sqrt{V}} |\mathbf{p}_A \mathbf{p}_B\rangle \frac{1}{\sqrt{2}} (|11\rangle + |22\rangle)$

$S_N(\rho^F_{\text{in}}) = 0,\ C(\rho^F_{\text{in}}) = 1$

post-scattering entanglement:

von Neuman entropy

$$\theta_1 = 1 - \Delta\,(1 - 16\pi\Delta)\,\frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{4\pi}$$

$$\theta_2 = \Delta\,(1 - 16\pi\Delta)\,\frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{4\pi}$$

S>0, entanglement increases

concurrence

$$C(\rho^F) = \sqrt{1 - \Delta\,(1 - 16\pi\Delta)\,\frac{(\lambda_1 - \lambda_2)^2 + (\lambda_6 + \lambda_7)^2}{2\,\pi}}$$

C<1, entanglement is reduced

Entanglement “flows”

from

flavor Hilbert space to full Hilbert space

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Unless:

- $\lambda_1 = \lambda_2,\ \lambda_6 = -\lambda_7$

Discrete 2HDM symmetries?
cf. Ferreira, Grządkowski, Ogreid, Osalnd *et al.* ‘23

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- $\Delta = \frac{1}{16\pi}$ — **Spherical symmetry of the initial wave packet (s wave)?**

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- $\Delta = \frac{1}{16\pi}$ — **Spherical symmetry of the initial wave packet (s wave)?**

Is conservation of entanglement related to a symmetry?
work in progress...

To take home

- Post-scattering entanglement may provide a **complementary way of constraining** the interaction structure of BSM models.

- Scattering interaction **injects** entanglement in a separable system, perturbatively small in λ, Δ .

- 2HDM: **no symmetry** from entanglement minimization.

- 2HDM: entanglement can be **transformed** by some coupling combinations, may lead to symmetries.