Massive quarks at one loop in the dipole picture of Deep Inelastic Scattering

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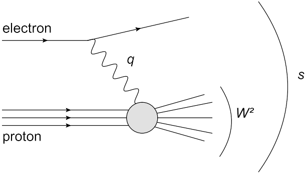
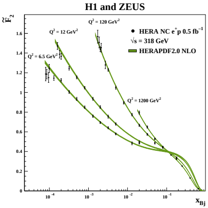
Deep inelastic scattering (DIS)





$$x_{Bj}\, Q^2\, \frac{d\sigma^{e+p\to e+X}}{dx_{Bj}\, d^2Q} = \frac{2\pi\, \alpha_{em}^2}{Q^2}\left[1+(1-y)^2\right]\Big\{F_T(x_{Bj},Q^2)+F_L(x_{Bj},Q^2) - \frac{y^2}{[1+(1-y)^2]}F_L(x_{Bj},Q^2)\Big\}$$

Photon virtuality : $Q^2 \equiv -q^2 > 0$

$F_2 = F_T + F_L$

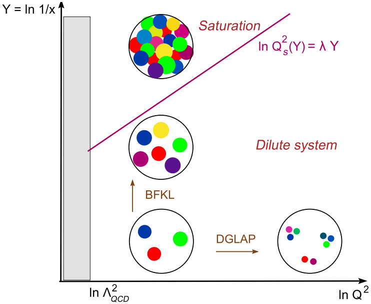
Bjorken variable : $x_{Bj} \equiv \frac{Q^2}{2P\cdot q} \sim \frac{Q^2}{W^2}$

Inelasticity : $y \equiv \frac{Q^2}{x_{Bj}\, s}$

Past : HERA at DESY. DIS on proton, with low luminosity.

Next decade : Electron-Ion-Collider (EIC) at BNL. DIS on proton and nuclei, with very high luminosity and the possibility of polarized beams.

Kinematical regimes of DIS



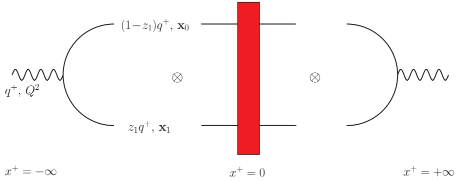
- For $Q^2 \to +\infty$: target more and more dilute due to DGLAP evolution.
  $\Rightarrow$ QCD-improved parton model more and more valid.
- For $x_{Bj} \to 0$: target more and more dense
  $\Rightarrow$ Linear BFKL evolution eventually breaks down, as well as parton picture.

  Onset of nonlinear collective effects: Gluon saturation!

Regime of large gluon field, but weak coupling α_s

Dipole factorization at high energy for DIS

At high energy ($x_{Bj} \to 0$) $\Rightarrow$ instantaneous interaction with the proton/nucleus target (Lorentz contraction)
$\Rightarrow$ Factorization of short-time and long-time dynamics



$$F_{T,L}(x_{Bj}, Q^2) = C_{T,L}(r, Q^2) \otimes N(r, Y) + O(\alpha_s) + O(x_{Bj})$$

avec $Y \sim \log(1/x_{Bj})$.

- Long time: $C_{T,L}(r, Q^2) \propto$ proba. of fluctuation of the photon into $q\bar{q}$ dipole, of size r, perturbatively calculable.
- Short time: $N(r, Y) =$ interaction proba. of the $q\bar{q}$ dipole with the proton/nucleus target (including gluon saturation effects)

High-energy Leading Log resummation: BK Equation

$N(r, Y)$ is non-perturbative, but its dependence in $Y \sim \log(1/x_{Bj}) \sim \log(W^2/Q^2)$ is perturbative:

Resummation of high-energy leading logs $\log(1/x_{Bj})$ (LL)

$\Rightarrow$ Evolution equation for $N(r, Y)$: (Balitsky-Kovchegov equation (BK))

$$\partial_Y N(r, Y) = \alpha_s K_1 \otimes N(r, Y) - \alpha_s K_2 \otimes N(r, Y) \otimes N(r, Y)$$

- Linear term (BFKL) $\Rightarrow$ fast growth at large Y (small x_{Bj})
- Nonlinear damping term $\Rightarrow$ $N(r, Y) \leq 1$: gluon saturation

Only needs an initial condition $N(r, Y = 0)$: fit on DIS data (now HERA, EIC in the future)

Dipole factorization : Universality

$N(r, Y)$ determines other observables

- In DIS at low x_{Bj} (HERA, EIC): diffractive, exclusive, semi-inclusive, ...
- In high energy proton-proton (pp) or proton-nucleus (pA) collisions (LHC, RHIC)

Ex: Single inclusive hadron production at forward rapidity y in pp or pA:

$$\frac{d\sigma}{dy\, d^2\mathbf{p}_\perp} = PDF \otimes N(r, Y) \otimes FF$$

Hybrid factorization : collinear and dipole

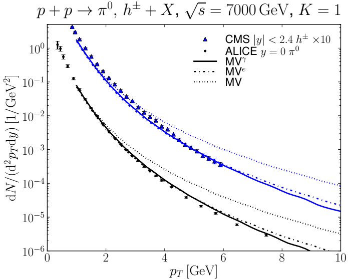
$N(r, Y)$ and gluon saturation physics also determine the dynamics of the earliest stage of relativistic heavy ion collisions, at LHC or RHIC

$\Rightarrow$ Initial conditions for the formation of the quark-gluon plasma

Gluon saturation phenomenology at LO+LL

- Fits of $N(r, Y)$ on DIS structure functions data from HERA, using the LO dipole factorisation and the BK equation
- Comparison of theory predictions using $N(r, Y)$ to LHC data

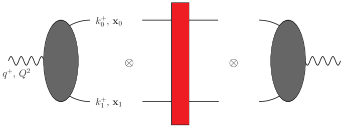
(Ex. Lappi, Mäntysaari (2013))

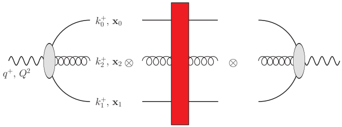


- State of the art: LO+LL precision → qualitatively OK, but lacks precision
- Not enough to prove (or falsify) the onset of nonlinear dynamics of gluon saturation

- Higher order corrections required!

NLO DIS calculation: massless quark case





$$F_{T,L}(Q^2, x_{Bj}) \propto \sum_{q\bar{q} \text{ states}} \left| \widetilde{\Psi}_{q\bar{q}}^{\gamma^*_{T,L}} \right|^2 N(\mathbf{x}_{01}, Y = 0)$$

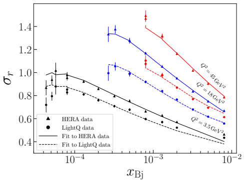
$$+ \sum_{q\bar{q}g \text{ states}} \left| \widetilde{\Psi}_{q\bar{q}g}^{\gamma^*_{T,L}} \right|^2 \Big[N(\mathbf{x}_{02}, 0) + N(\mathbf{x}_{21}, 0) - N(\mathbf{x}_{02}, 0)\, N(\mathbf{x}_{21}, 0) \Big] + O(\alpha_{em}\, \alpha_s^2)$$

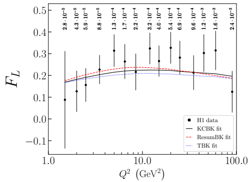
- Perturbative building blocks for NLO DIS:
  $\widetilde{\Psi}_{q\bar{q}}^{\gamma^*_{T,L}}$ LFWF at one loop and $\widetilde{\Psi}_{q\bar{q}g}^{\gamma^*_{T,L}}$ LFWF at tree-level
- UV divergences shown to cancel between $q\bar{q}$ and $q\bar{q}g$ ($\to$ Dim. Reg.)
- High-energy LL resummation at the end : BK evolution to appropriate $Y > 0$

G.B. (2016-2017) & Hänninen, Lappi and Paatelainen (2017)

see also Balitsky and Chirilli (2011-2013)

NLO DIS fit with massless quarks





Fits of $N(r, Y)$ using massless NLO dipole cross section and BK equation at LL (with collinear resummations)

- On HERA data for σ_r ($\simeq F_2$)
- On an estimation of the light quark contribution in the data

G.B., Hänninen, Lappi, and Mäntysaari (2020)

- Successful individual fits, including prediction of F_L
- But very different values for fit parameters obtained from full data or from the light quark contribution

Massive quarks known to give a sizable contribution to DIS
$\Rightarrow$ NLO corrections in the massive quark case also necessary for the desired precision

Massive quarks in low x DIS at one loop

- F_L at NLO with quark masses:
  G.B., Lappi and Paatelainen, Phys.Rev.D 104 (2021) 5, 056032
- F_T at NLO with quark masses:
  G.B., Lappi and Paatelainen, arXiv:2112.03158 [hep-ph]

$$F_{T,L}(Q^2,x_{Bj}) \propto \sum_{q\bar{q}\text{ states}} \left|\widetilde{\Psi}_{q\bar{q}}^{\gamma^*_{T,L}}\right|^2 N(\mathbf{x}_{01},Y{=}0)$$
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Same general structure/method in the massive quark case as in the massless case, but significantly lengthier, and with extra complications:

Conceptual : Quark mass renormalization necessary at this order, but until now mass renormalization was not formulated in a consistent way in Light-Front perturbation theory.

Technical :
- Extra contributions in the massive case due to helicity flips
- Appearance of integrals defining new special functions (generalizations of Bessel functions)

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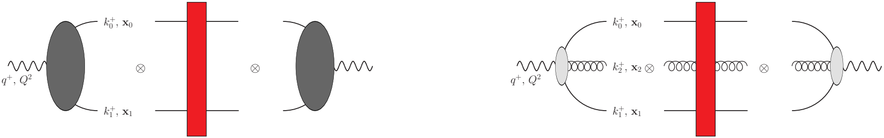

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Conclusion

- NLO corrections including full quark mass dependence have been obtained for the DIS structure functions in the dipole factorization at low x_{Bj}.
- All the ingredients are now available to perform precise and reliable NLO fits of $N(r, Y)$

  $\Rightarrow$ Will allow precise predictions for other low x_{Bj} observables at EIC or LHC
- Crucial milestone towards the quantitative understanding of the nonlinear gluon saturation regime in protons and nuclei.
- Methods developed for that calculation and even intermediate results will prove useful for the NLO calculation of other observables.