Probing an ultrarelativistic heavy ion at next-to-eikonal accuracy

Alina Czajka

BP2, Department of Fundamental Research, NCBJ

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EIC in the focus of QCD group

Goals of Electron-Ion Collider - one of the main activities of the QCD group.

EIC is expected to start operating in the next decade.

Among its main goals:

- 3D momentum distribution of partons inside protons or nuclei - transverse momentum dependent distributions (**TMDs**) and generalized parton distributions (**GPDs**)

  (L. Szymanowski, J. Wagner, P. Sznajder, V. Martinez-Fernandez)

- saturation effects inside protons or nuclei - phenomenon predicted by Color Glass Condensate (**CGC**)

  (T. Altinoluk, G. Beuf, A. Czajka, P. Agostini, E. Blanco, A. Tymowska (graduated), S. Nisar Mulani)

Back-to-back DIS dijet production

Consistency and interplay between CGC and TMD factorization formalisms?

For a process with two transverse momentum scales: $\mathbf{P}$ (hard) and $\mathbf{k}$ (not so hard):

- CGC result: leading power (eikonal) in the limit $|\mathbf{k}| \sim |\mathbf{P}| \ll \sqrt{s}$
- TMD factorization: leading power (twist-2) in the limit $|\mathbf{k}| \ll |\mathbf{P}| \sim \sqrt{s}$

⋆ eikonal and twist-2 correlator $\langle \mathcal{F}_i{}^{-} \mathcal{F}_j{}^{-} \rangle$

$\Rightarrow$ What about power corrections in $\mathbf{P}^2/s$ or $|\mathbf{P}||\mathbf{k}|/s$ beyond the eikonal limit?

$\Rightarrow$ What about power corrections in $|\mathbf{k}|/|\mathbf{P}|$ (kinematical higher twist contributions)?

$\Rightarrow$ What about genuine higher twist, beyond-eikonal corrections (correlators involving other combinations of the field strength components)?

(Altinoluk, Beuf, Czajka, Marquet, to appear)

Eikonal approximation in the CGC

In the CGC framework two approximations adopted:

(i) Semi-classical approximation → dense target is represented by a **strong semi-classical gluon field** $\mathcal{A}^\mu(x)$

(ii) Eikonal approximation → can be understood as the limit of **infinite boost** of $\mathcal{A}^\mu(x)$:

- Under a boost of parameter γ_t along the "$-$" direction, strong ordering between the components of the field:

$$\mathcal{A}^- = O(\gamma_t) \;\gg\; \mathcal{A}_\perp = O(1) \;\gg\; \mathcal{A}^+ = O(1/\gamma_t)$$

  ⋆ Only the enhanced component of the background field ($\mathcal{A}^-$) is kept.

- Lorentz contraction of the background field $\mathcal{A}^\mu(x)$ (shockwave limit)

  ⋆ background field is localized around $x^+ = 0$ (no transverse motion within the target)

- $\mathcal{A}^\mu(x)$ independent on x^- (static limit) due to Lorentz time dilation

  ⋆ dynamics of the target is neglected (no p^+ transfer from the target)

Background field in the eikonal limit: $\mathcal{A}^\mu(x^+, x^-, \mathbf{x}) \approx \delta^{\mu -} \mathcal{A}^-(x^+, \mathbf{x}) \propto \delta(x^+)$

Eikonal interaction between the projectile and the target:

- each parton picks up a Wilson line during the interaction

$$U_\mathcal{R}(\mathbf{x}) = \mathcal{P}_+ \exp\left[ig \int dx^+ T^a_\mathcal{R} A^-_a(x^+, \mathbf{x})\right]$$

- dipole operator appears in the observable

$$d_\mathcal{R}(\mathbf{x}, \mathbf{y}) = \frac{1}{D_\mathcal{R}} \mathrm{tr}\left[U_\mathcal{R}(\mathbf{x}) U^\dagger_\mathcal{R}(\mathbf{y})\right]$$

Next-to-Eikonal corrections to the CGC

Next-to-Eikonal (NEik) power corrections to the standard CGC formalism:

- Of order $1/\gamma_t$ at the level of the boosted background field
- Of order $1/s$ at the level of a cross section

NEik corrections arise from relaxing either of the three approximations:

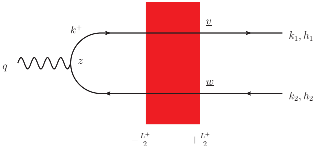
1. Interactions with $\mathcal{A}_\perp$ field taken into account, not only $\mathcal{A}^-$
2. Target with finite longitudinal width $\Rightarrow$ transverse motion of the parton within the target
3. x^- dependence of $\mathcal{A}^\mu(x)$ beyond infinite Lorentz dilation

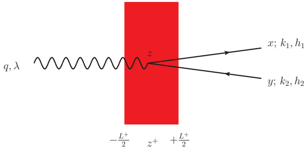
   $\Rightarrow$ treated as gradient expansion around a common x^- value:

$$\boxed{\frac{\partial_- \mathcal{A}^-(x)}{\mathcal{A}^-(x)} = O(1/\gamma_t)}$$

   $\Rightarrow$ allows possibility of (small) p^+ exchange with the target

DIS dijet at NEik accuracy: S-matrix for γ_L^*





DIS dijet cross section calculated at NEik accuracy, at LO in α_s in the CGC (Altinoluk, Beuf, Czajka, Tymowska, 2023)

- Only longitudinal photon contribution will be discussed for simplicity
- Second diagram vanishes in γ_L^* case, but matters in γ_T^* case

S-matrix element at NEik accuracy (longitudinal photon polarization)

$$S_{q_1\bar{q}_2\leftarrow\gamma_L^*} = S^{\text{Gen.Eik}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dyn.target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} + S^{\text{dec. on} \bar{q}}_{q_1\bar{q}_2\leftarrow\gamma_L^*}$$

with

$$S^{\text{Gen.Eik}}_{q_1\bar{q}_2\leftarrow\gamma^* L} = -2Q\,\frac{ee_f}{2\pi}\,\bar{u}(1)\gamma^+ v(2)\,\frac{(q^+ + k_1^+ - k_2^+)(q^+ + k_2^+ - k_1^+)}{4(q^+)^2}\int_{\mathbf{v},\mathbf{w}} e^{-i\mathbf{v}\cdot\mathbf{k}_1}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times\, \mathrm{K}_0\left(\hat{Q}\,|\mathbf{w}-\mathbf{v}|\right)\int db^-\, e^{ib^-(k_1^+ + k_2^+ - q^+)}\left[\mathcal{U}_F\left(\mathbf{v}, b^-\right)\mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right) - 1\right]$$

⋆ 0th order term in the expansion around a common value $b^- = (v^- + w^-)/2$

⋆ resembles the strict eikonal term with extra b^- dependence

DIS dijet at NEik accuracy

$$S^{\text{dyn. target}}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, iQ\, \frac{ee_f}{2\pi}\, \bar{u}(1)\gamma^+ v(2)\, \frac{(k_1^+ - k_2^+)}{(q^+)^2} \int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1} \int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}$$
$$\times \left[\mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right) - \frac{\left(\bar{Q}^2 - m^2\right)}{2\bar{Q}}\, |\mathbf{w}-\mathbf{v}|\, \mathrm{K}_1\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right) \right] \left[\mathcal{U}_F\left(\mathbf{v}, b^-\right) \overleftrightarrow{\partial_{b^-}} \mathcal{U}_F^\dagger\left(\mathbf{w}, b^-\right) \right]\Big|_{b^-=0}$$

⋆ first term in the expansion of the around the common value b^-

$$S^{\text{dec. on } q}_{q_1\bar{q}_2\leftarrow\gamma_L^*} = 2\pi\delta(k_1^+ + k_2^+ - q^+)\, \frac{ee_f}{2\pi}\, (-1)Q\, \frac{k_2^+}{(q^+)^2} \int d^2\mathbf{v}\, e^{-i\mathbf{v}\cdot\mathbf{k}_1} \int d^2\mathbf{w}\, e^{-i\mathbf{w}\cdot\mathbf{k}_2}\, \mathrm{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\ \bar{u}(1)\gamma^+ \left[\frac{[\gamma^i, \gamma^j]}{4}\, \mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) - i\,\mathcal{U}^{(2)}_F(\mathbf{v}) + \mathcal{U}^{(1)}_{F;j}(\mathbf{v}) \left(\frac{(\mathbf{k}_2^j - \mathbf{k}_1^j)}{2} + \frac{i}{2}\,\partial_{\mathbf{w}^j} \right) \right] \mathcal{U}_F^\dagger(\mathbf{w})\, v(2)$$

⋆ similar expression for $\bar{q}$

⋆ stem from finite width and the interaction with the transverse component of the background field

<u>decorated Wilson lines:</u>

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right) \overleftrightarrow{\mathcal{D}_{\mathbf{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)$$

$$\mathcal{U}^{(2)}_{F}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right) \overleftarrow{\mathcal{D}_{\mathbf{v}^j}}\, \overrightarrow{\mathcal{D}_{\mathbf{v}^j}} \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)$$

$$\mathcal{U}^{(3)}_{F;ij}(\mathbf{v}) = \int_{-\frac{L^+}{2}}^{\frac{L^+}{2}} dv^+\, \mathcal{U}_F\left(\frac{L^+}{2}, v^+; \mathbf{v}\right) gt\cdot\mathcal{F}_{ij}(\underline{v}) \mathcal{U}_F\left(v^+, -\frac{L^+}{2}; \mathbf{v}\right)$$

NEik corrections as $\mathcal{F}^{\mu\nu}$ insertions

Expression for the cross section is lengthy before taking the back-to-back limit!
Consider only one term of the cross section to discuss the idea!

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\Bigg|^{\text{dec. on } q}_{\text{NEik corr.}} = (2q^+)\, 2\pi\delta(k_1^+ + k_2^+ - q^+)\; 8k_1^+k_2^+Q^2 \left(\frac{ee_f}{2\pi}\right)^2 \frac{k_1^+k_2^+}{(q^+)^3}\,\frac{k_2^+}{2(q^+)^3}$$
$$\times\, 2\text{Re}\int_{\mathbf{v},\mathbf{v}',\mathbf{w},\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{v})}\, e^{i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{w})}\text{K}_0\left(\bar{Q}\,|\mathbf{w}'-\mathbf{v}'|\right)\text{K}_0\left(\bar{Q}\,|\mathbf{w}-\mathbf{v}|\right)$$
$$\times\, \text{Tr}\Big\langle\Big[\mathcal{U}_F(\mathbf{w}')\mathcal{U}_F^\dagger(\mathbf{v}') - 1\Big]\Big[\Big(-i\,\mathcal{U}_F^{(2)}(\mathbf{v}) + \frac{(\mathbf{k}_2^j-\mathbf{k}_1^j)}{2}\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\Big)\mathcal{U}_F^\dagger(\mathbf{w}) + \frac{i}{2}\mathcal{U}_{F;j}^{(1)}(\mathbf{v})\;\partial_{\mathbf{w}^j}\mathcal{U}_F^\dagger(\mathbf{w})\Big]\Big\rangle$$

Terms with $\mathcal{U}_{F;ij}^{(3)}(\mathbf{v})$ cancel at cross section level for γ_L^*, but survive for γ_T^*

On the way to TMDs: the relation between derivatives of the Wilson lines and field strength insertions:

$$\partial_\mu\mathcal{U}_F(x^+,y^+;\mathbf{v},v^-) + igt\cdot\mathcal{A}_\mu(x^+,\mathbf{v},v^-)\mathcal{U}_F(x^+,y^+;\mathbf{v},v^-) - ig\mathcal{U}_F(x^+,y^+;\mathbf{v},v^-)t\cdot\mathcal{A}_\mu(y^+,\mathbf{v},v^-)$$
$$= -ig\int_{y^+}^{x^+} dv^+\mathcal{U}_F(x^+,v^+;\mathbf{v},v^-)t\cdot\mathcal{F}_\mu^{\ -}(v)\mathcal{U}_F(v^+,y^+;\mathbf{v},v^-) \qquad \text{for } \mu\neq +$$

Change of variables and back-to-back limit

In momentum space:

(total dijet momentum) $\mathbf{k} = \mathbf{k}_1 + \mathbf{k}_2$ and (relative momentum) $\mathbf{P} = (z_2\mathbf{k}_1 - z_1\mathbf{k}_2)$

$z_1 = k_1^+/(k_1^+ + k_2^+)$ and $z_2 = k_2^+/(k_1^+ + k_2^+) = 1 - z_1$ such that

In coordinate space:

(conjugate to $\mathbf{k}$) $\mathbf{b} = (z_1\mathbf{v} + z_2\mathbf{w})$ and (conjugate to P) $\mathbf{r} = \mathbf{v} - \mathbf{w}$

back-to-back correlation limit: $|\mathbf{k}| \ll |\mathbf{P}|$ and $|\mathbf{r}| \ll |\mathbf{b}|$

perform a small $\mathbf{r}$ expansion at the level of the squared amplitude and go to adjoint representation

DIS dijet production cross section at NEik accuracy written in terms of field strength insertions!

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\mathcal{U}^{\dagger}_F\Big(\mathbf{b}\Big) = 2it^{a'}\int_{z^+} z^+\,\mathcal{U}_A\Big(+\infty, z^+;\mathbf{b}\Big)_{a'a}\, g\mathcal{F}_j^{a\,-}(z^+,\mathbf{b})$$

$$\mathcal{U}^{(2)}_{F}(\mathbf{b})\mathcal{U}^{\dagger}_F(\mathbf{b}) = -t^{a'}t^{b'}\int_{z^+,z'^+} (z^+ - z'^+)\,\theta(z^+ - z'^+)\mathcal{U}_A\Big(+\infty, z^+;\mathbf{b}\Big)_{a'a}\, g\mathcal{F}_j^{a\,-}(z^+,\mathbf{b})\mathcal{U}_A\Big(+\infty, z'^+;\mathbf{b}\Big)_{b'b}\, g\mathcal{F}_j^{b\,-}(z'^+,\mathbf{b})$$

$$\mathcal{U}^{(1)}_{F;j}(\mathbf{b})\,\partial_i\mathcal{U}^{\dagger}_F(\mathbf{b}) = -2t^{a'}t^{b'}\int_{z^+,z'^+} z^+\,\mathcal{U}_A(+\infty, z^+;\mathbf{b})_{a'a}\, g\mathcal{F}_j^{a\,-}(z^+,\mathbf{b})\mathcal{U}_A(+\infty, z'^+;\mathbf{b})_{b'b}\, g\mathcal{F}_i^{b\,-}(z'^+,\mathbf{b})$$

⋆ **contributions with either 1 or 2 $\mathcal{F}_\perp^{\,-}$ (like in generalized eikonal case)**
⋆ **now with an extra factor z^+ or $(z^+ - z'^+)$ (NEik suppression with the target width)**
→ similar results for decorations on the antiquark line

Back-to-back cross section: Eikonal piece

The dijet cross section for the longitudinal photon in the back-to-back correlation limit:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}} = \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{Eik}} + \frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\text{corr. lim.}}_{\text{NEik corr.}}$$

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}}_{\text{Eik}} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4 z_1^3 z_2^3 Q^2$$
$$\times\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j + \mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left\langle \mathcal{F}_i^{a\;-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\;-}(z^+,\mathbf{b})\right\rangle$$

- **twist-2 gluon TMDs in the target** (both unpolarized and linearly polarized), with momentum fraction $x = 0$ and transverse momentum $\mathbf{k}$, with a *future staple* gauge link
- **kinematical twist-3 corrections**, suppressed by an extra $|\mathbf{k}|/|\mathbf{P}|$ in the back-to-back dijet limit $|\mathbf{k}| \ll |\mathbf{P}|$
- not shown here: **genuine twist-3 corrections**, involving a correlator of the type $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\rangle$
- difference between Gen. Eik and Strict Eik.: involves correlator $\langle \mathcal{F}_\perp^{\;-}\mathcal{F}_\perp^{\;-}\mathcal{F}^{+-}\rangle \Rightarrow$ twist-4 and NEik correction!

Back-to-back cross section: twist-3 TMDs from NEik

From the interference between the non-static NEik correction and the strict Eikonal amplitudes:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{NEik}^{\mathcal{F}_\perp^-\mathcal{F}^{+-}} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)8Q^2e^2e_f^2g^2\,\frac{z_1^2z_2^2(z_2-z_1)}{q^+}\,\frac{\mathbf{P}^i(\mathbf{P}^2+m^2)}{(\mathbf{P}^2+\bar{Q}^2)^4}$$
$$\times\, 2\mathrm{Re}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\Big\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_b^{+-}(z^+,\mathbf{b})\Big\rangle$$

⇒ NEik. correction stemming from the dynamics of the target is a **twist-3 gluon TMD**, (Mulders, Rodrigues, 2001) with momentum fraction $\mathrm{x}=0$.

From the interference between the NEik correction with $\mathcal{U}_{F;ij}^{(3)}$ and the strict Eikonal amplitude:

- Vanishing result in the γ_L^* case due to Dirac algebra.
- An extra contribution to the cross section in the γ_T^* case:

$$\frac{d\sigma_{\gamma_T^*\to q_1\bar{q}_2}}{d\mathrm{P.S.}}\Bigg|_{NEik}^{\mathcal{F}_\perp^-\mathcal{F}_{ij}} \propto 2\mathrm{Re}\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\Big\langle \mathcal{F}_l^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(\infty,z'^+;\mathbf{b}')\mathcal{U}_A(\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_{ij}^b(z^+,\mathbf{b})\Big\rangle$$

⇒ The other **twist-3 gluon TMD** as found in Mulders, Rodrigues, 2001, with momentum fraction $\mathrm{x}=0$.

Back-to-back cross section: x dependence from NEik

Including all contributions of the form $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$, of order Eik or NEik, and twist-2 or twist-3:

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4z_1^3 z_2^3 Q^2$$
$$\times\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+}\left[1 + i(z^+ - z'^+)\frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+z_1z_2}\right]\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$

$\Rightarrow$ NEik corrections and kinematic twist-3 corrections to the $\langle \mathcal{F}_\perp^- \mathcal{F}_\perp^- \rangle$ contribution factorize from each other!

The " $-$ " momentum extracted from the target can be defined from the conservation relation (where the $\mathbf{k}^2$ term is a twist-4 correction):

$$\text{x}P^-_{tar.} \equiv \check{k}_1^- + \check{k}_2^- - q^- = \frac{\mathbf{k}_1^2+m^2}{2k_1^+} + \frac{\mathbf{k}_2^2+m^2}{2k_2^+} + \frac{Q^2}{2q^+} = \frac{(\mathbf{P}^2+\bar{Q}^2)}{2q^+z_1z_2} + \frac{\mathbf{k}^2}{2q^+}$$

The NEik correction can be summed into a phase! $\Rightarrow$ dependence of the twist-2 gluon TMDs on x

$$\frac{d\sigma_{\gamma_L^*\to q_1\bar{q}_2}}{d\text{P.S.}}\bigg|^{\mathcal{F}_\perp^-\mathcal{F}_\perp^-} = (2q^+)2\pi\delta(k_1^+ + k_2^+ - q^+)(ee_f)^2 g^2 4z_1^3 z_2^3 Q^2$$
$$\times\left[\frac{4\mathbf{P}^i\mathbf{P}^j}{(\mathbf{P}^2+\bar{Q}^2)^4} - 2(z_2-z_1)\frac{(\mathbf{P}^i\mathbf{k}^j+\mathbf{k}^i\mathbf{P}^j)}{[\mathbf{P}^2+\bar{Q}^2]^4} + 16(z_2-z_1)\frac{(\mathbf{k}\cdot\mathbf{P})\mathbf{P}^i\mathbf{P}^j}{[\mathbf{P}^2+\bar{Q}^2]^5} + O\left(\frac{\mathbf{k}^2}{\mathbf{P}^8}\right)\right]$$
$$\times\int_{\mathbf{b},\mathbf{b}'} e^{-i\mathbf{k}\cdot(\mathbf{b}-\mathbf{b}')}\int_{z^+,z'^+} e^{i(z^+-z'^+)\text{x}P^-_{tar.}}\left\langle \mathcal{F}_i^{a\,-}(z'^+,\mathbf{b}')\left[\mathcal{U}_A^\dagger(+\infty,z'^+;\mathbf{b}')\mathcal{U}_A(+\infty,z^+;\mathbf{b})\right]_{ab}\mathcal{F}_j^{b\,-}(z^+,\mathbf{b})\right\rangle$$

(Altinoluk, Beuf, Czajka, Marquet, to appear)

Summary

To understand the interplay between CGC and TMD frameworks, we studied the back-to-back limit of the DIS dijet production at NEik accuracy, including twist-3 power corrections.

We obtained various contributions:

- twist-2 gluon TMDs: $\langle \mathcal{F}_i^{\ -} \mathcal{F}_j^{\ -} \rangle$
  - factorization of kinematic twist-3 and of NEik correction
  - NEik corrections reproduce the expansion of the phase defining the x dependence of the TMDs
- twist-3 gluon TMDs: $\langle \mathcal{F}_i^{\ -} \mathcal{F}^{+-} \rangle$ and (for γ_T^*) $\langle \mathcal{F}_l^{\ -} \mathcal{F}_{ij} \rangle$ as further NEik corrections
- 3-body twist-3 correlators $\langle \mathcal{F}_i^{\ -} \mathcal{F}_j^{\ -} \mathcal{F}_l^{\ -} \rangle$: beyond TMDs!
  Already appear in Eikonal contributions. NEik corrections partially resum into phase.