Curvature of the Universe

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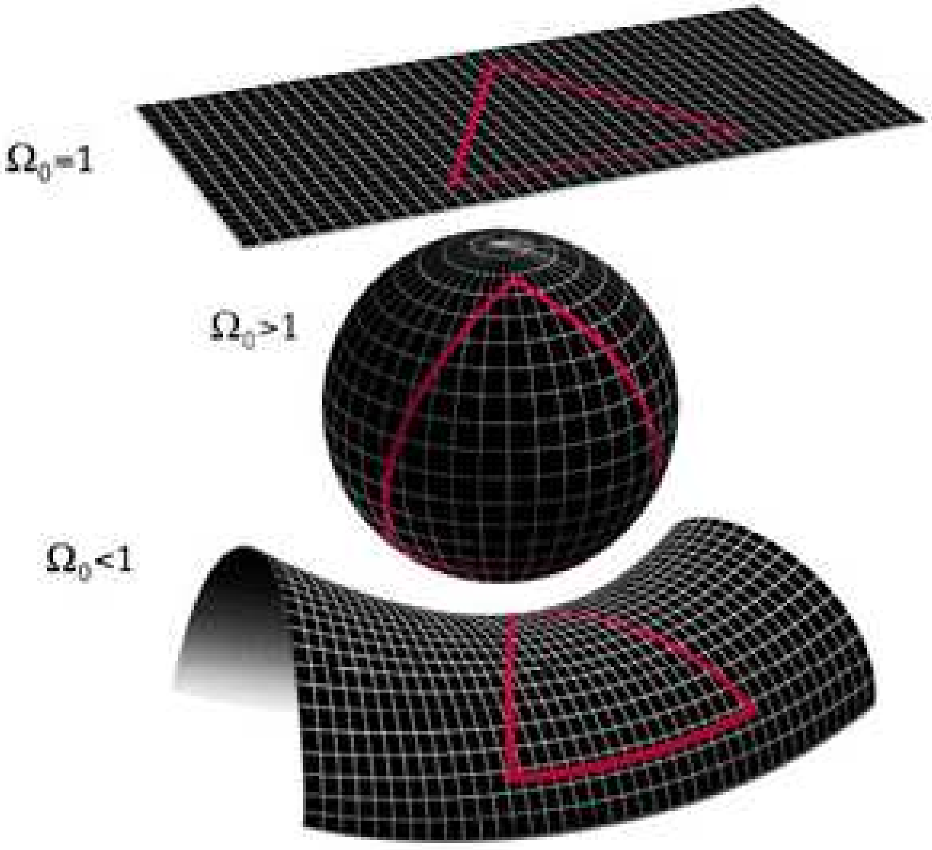
Spatial curvature in the FLRW universe

Cosmic triangle: $1 = \Omega_\mathrm{m} + \Omega_\mathrm{k} + \Omega_\Lambda$

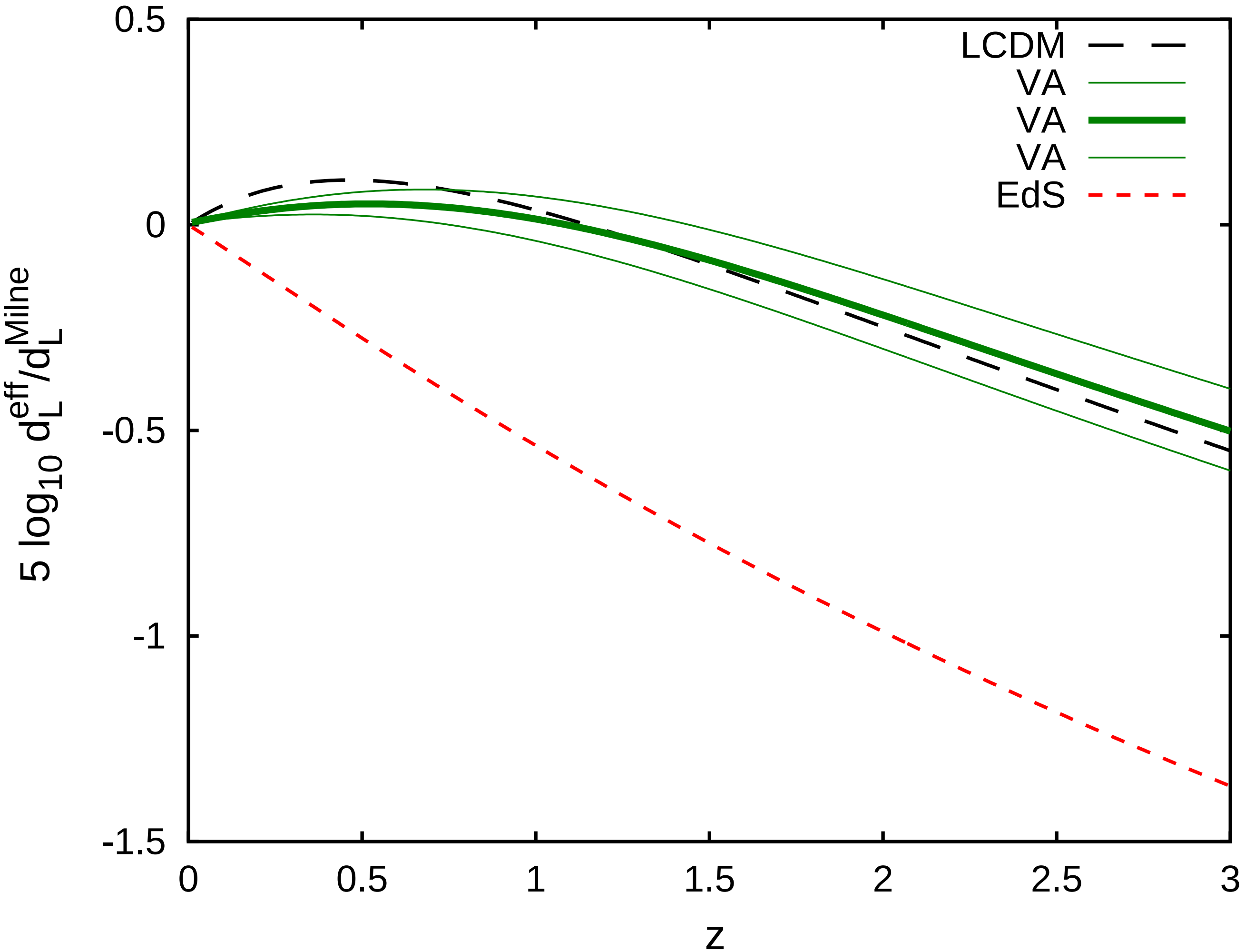


Buchert, Ellis and van Elst, Gen. Rel. and Grav. 2009

We think that these order–of–magnitude estimates provide a strong call for a proper relativistic treatment of the underlying gravitational physics in these systems; spatial curvature is an inherently relativistic phenomenon, unknown to the Newtonian theory. The claim on the validity of a quasi–Newtonian metric (..) *to describe gravitational physics on all scales in the observable Universe* (...) *is thus seriously called into question.*

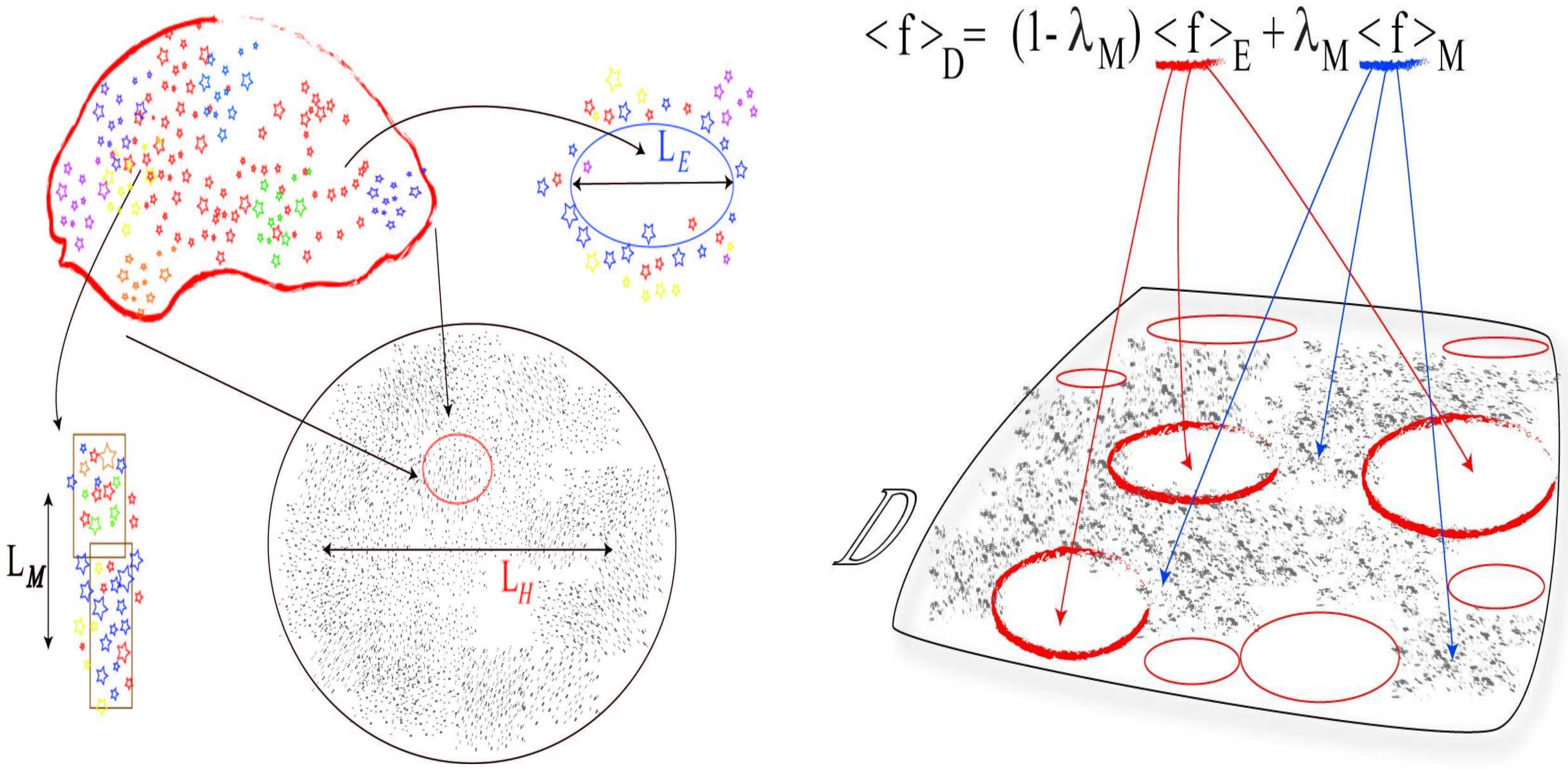
Gravitating system / Smoothing scale	Mass M	Diameters D and d	D/d	ε	$\varepsilon(D/d)^2$
A1: Earth's orbit / Sun	$\approx M_\odot$ (1.99×10^{30} kg)	300×10^6 km 1.39×10^6 km	216	4.24×10^{-6}	0.20
A2: Galaxy / Open star cluster	$\approx 10^{11}\ M_\odot$ (1.99×10^{41} kg)	100000 ly 30 ly	3333	6.23×10^{-7}	6.92
A3: Cluster of galaxies / Galaxy	$\approx 10^{14}\ M_\odot$ (1.99×10^{44} kg)	5 Mpc 0.03 Mpc	167	3.82×10^{-6}	0.11
C1: Void / Wall	$\approx (1/6)\pi\rho_m D^3$ (2.98×10^{45} kg)	$30h^{-1}$ Mpc $3h^{-1}$ Mpc	10	6.78×10^{-6}	6.78×10^{-4}
C2: Homogeneity scale / Supercluster	$\approx (1/6)\pi\rho_m D^3$ (2.98×10^{48} kg)	$300h^{-1}$ Mpc $30h^{-1}$ Mpc	10	6.78×10^{-4}	6.78×10^{-2}
C3: Hubble sphere / —	$\approx (1/6)\pi\rho_m D^3$ (2.38×10^{52} kg)	$6000h^{-1}$ Mpc —	—	0.27	—

Spatial curvature: observational effect



Roukema, Ostrowski, Buchert; JCAP 2013

Partitioning approach



Buchert, Carfora; CQG 2002

Hamiltonian constraint, turnaround condition

Local Hamiltonian constraint:

$$3H^2 = 8\pi G\rho - 3k/a^2 + \Lambda \quad \Rightarrow \quad H^2 = 8\pi G\rho + \sigma^2 - \frac{1}{2}\mathcal{R} + \Lambda$$

Turnaround condition: $H = 0$

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Averaged Hamiltonian constraint:

$$H_{\mathcal{D}}^2 = 8\pi G\langle\rho\rangle_{\mathcal{D}} - \frac{1}{2}\mathcal{Q} - \frac{1}{2}\langle\mathcal{R}\rangle_{\mathcal{D}} + \Lambda$$

where $\mathcal{Q}$ contains kinematical effects from inhomogeneities

Turnaround condition: $H_{\mathcal{D}} = 0$

Analytical results

For the turnaround to occur:

$$\mathcal{R} > 0$$

In the case of averaged equations, we have statistically:

$$\langle \mathcal{R} \rangle_{\mathcal{D}} > 0$$

For Einstein de-Sitter background we have:

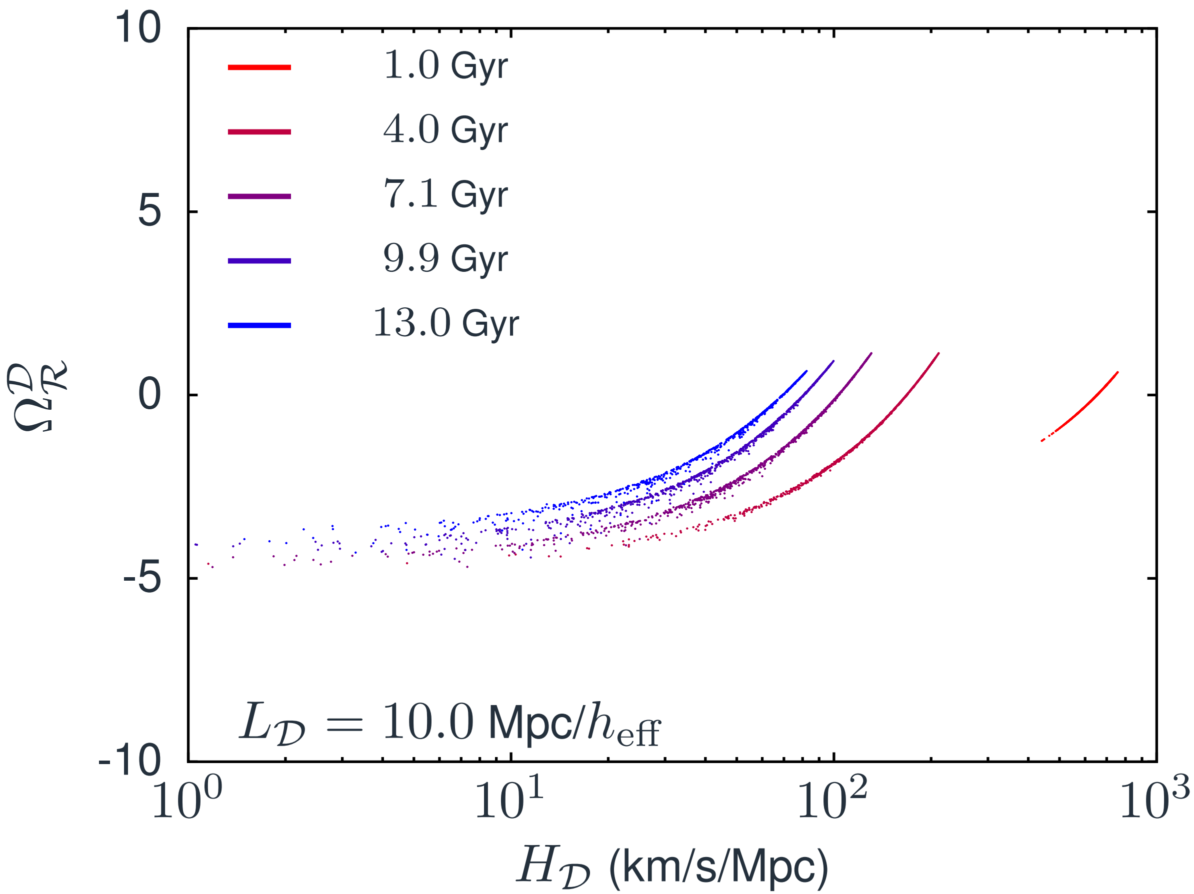
$$\Omega_{\mathcal{R}}^{\mathcal{D}} = -5 \;\; ; \;\; \Omega_{\mathcal{Q}}^{\mathcal{D}} = 1 \;\; ; \;\; \Omega_{m}^{\mathcal{D}} = 4 \;\; ; \;\; \frac{\langle \rho \rangle_{\mathcal{D}}}{\rho_{EdS}} = 4$$

Numerical methods

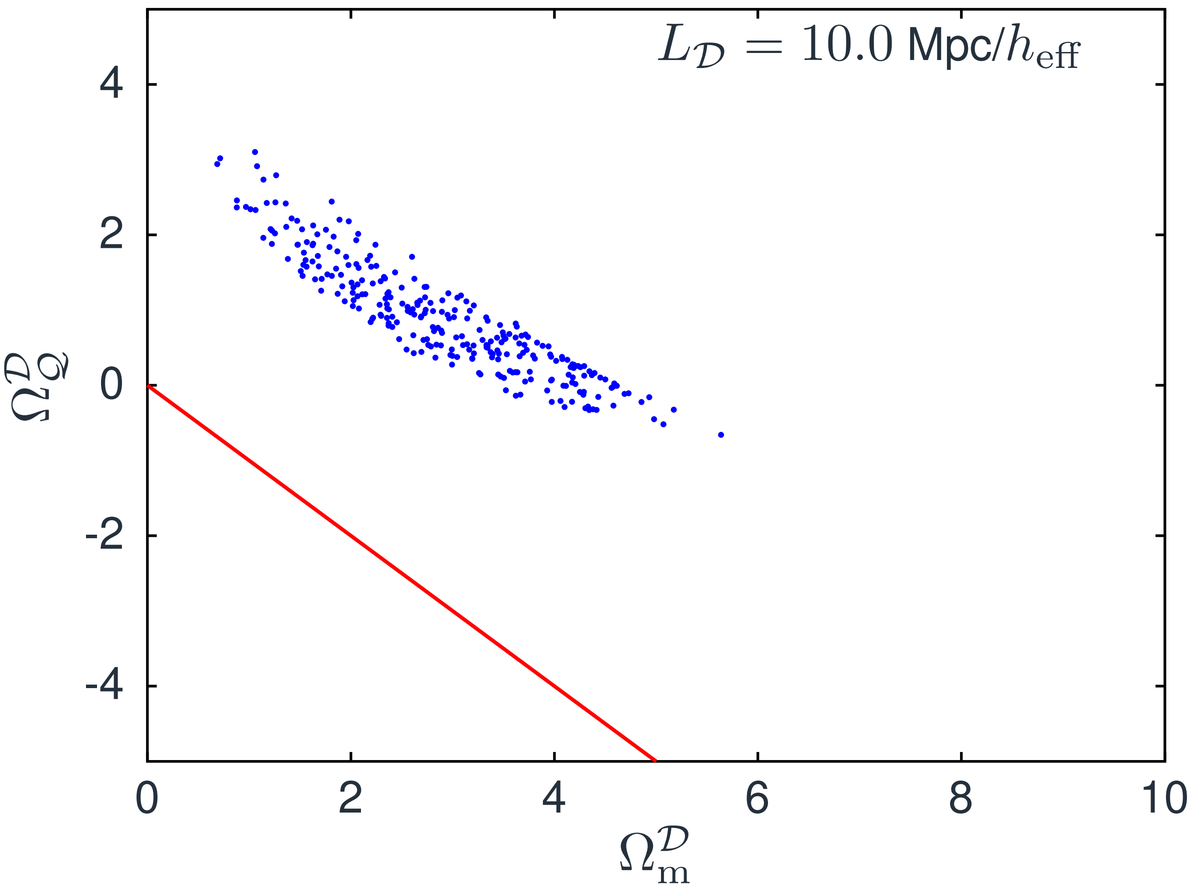
General scheme:

- **MPGRAFIC** - generate initial conditions
- **DTFE** - calculate averaged initial conditions
- **INHOMOG** - calculate evolution of the domains
- **RAMSES-SCALAV** single pipeline + additional options

Curvature density



Averaged positive curvature



Conclusions

- big positive spatial curvature is a generic feature of collapsing structures at the turnaround; both locally and on average
- $\Omega_{\mathcal{R}}^{\mathcal{D}} \approx -5$ remains an approximate lower bound for the averaged curvature functional for the wide range of initial conditions
- fluid parameters at the turnaround may provide an additional cosmological test
- details can be found in:

$\rightarrow$ **'A few numbers from the turnaround epoch of collapse', Ostrowski J.J., Acta Phys. Pol. B Proc. Suppl. Vol. 13, p 177 (2020)**

$\rightarrow$ **'Does spatial flatness forbid the turnaround epoch of collapsing structures?', Roukema B.F., Ostrowski J.J., Journal of Cosmology and Astroparticle Physics, Vol. 2019, 12 (2019)**