# Properties of heaviest nuclei

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NARODOWE CENTRUM BADAŃ JĄDROWYCH ŚWIERK



### Properties of heaviest nuclei with $98 \le Z \le 126$ and $134 \le N \le 192$

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#### ARTICLE INFO

#### ABSTRACT

Article history: Received 21 October 2020 Accepted 5 November 2020 Available online xxxx We systematically determine ground-state and saddle-point shapes and masses for 1305 heavy and superheavy nuclei with Z = 98-126 and N = 134-192, including odd-A and odd-odd systems. From these we derive static fission barrier heights, one- and two-nucleon separation energies, and  $O_{\alpha}$  values for g.s. to g.s transitions. Our study is performed within the microscopic-macroscopic method with the deformed Woods-Saxon single-particle potential and the Yukawa-plus-exponential macroscopic energy taken as the smooth part. We use parameters of the model that were fitted previously to masses of even-even heavy nuclei. For systems with odd numbers of protons, neutrons, or both, we use a standard BCS method with blocking. Ground-state shapes and energies are found by the minimization over seven axially-symmetric deformations. A search for saddle-points was performed by using the "imaginary water flow" method in three consecutive stages, using five- (for nonaxial shapes) and seven-dimensional (for reflection-asymmetric shapes) deformation spaces. Calculated ground-state mass excess, nucleon separation- and  $Q_{\alpha}$  energies, total, macroscopic (normalized to the macroscopic energy at the spherical shape) and shell corrections energies, and deformations are given for each nucleus in Table 1. Table 2 contains calculated properties of the saddle-point configurations and the fission barrier heights. In Tables 3-7, are given calculated ground-state, inner and outer saddle-point and superdeformed secondary minima characteristics for 75 actinide nuclei, from Ac to Cf, for which experimental estimates of fission barrier heights are known. These results are an additional test of our model.

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Nuclear masses: M and Q-values



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Fission barriers & differences between fission barriers and neutron separation energies





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#### Static fission properties of actinide nuclei

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- ✓ We have systematically determined inner and outer fission barrier heights for 75 actinides, within the range from actinium to californium, including odd-A and odd-odd systems, for which experimental estimates were accessible.
- $\checkmark$  A statistical comparison of our fission barrier heights with available experimental estimates gives the average discrepancy and the rms deviation not greater than 0.82 and 0.94 MeV, respectively. This concerns both first and second fission barriers.
- Determined excitation energies of superdeformed secondary minima reproduce quite well the general rends of experimental data. The largest discrepancies do not exceed 1.1 MeV. There is also an intriguing question of third minima, which in our calculations, if they appear at all, are rather shallow: in most cases they do not exceed 0.5–0.6 MeV in depth.



TABLE II. Statistical parameters of the comparison of our first fission barrier heights  $B_f^{(l)th}$  with experimental estimates taken from [47,48]. The average discrepancy  $\overline{\Delta}$  and the rms deviation  $\delta_{\rm rms}$  are in MeV, and N is the number of considered nuclei.

	Comparison for $Z = 90-98$		
	$B_{f}^{(l)th}$ vs EXP1 [47]	$B_{f}^{(I)\text{th}}$ vs EXP2 [48]	
	71	45	
	0.80	0.73	
15	0.94	0.85	

N

Ā

Ā

TABLE III. The same as in Table II but for our second fission barrier heights  $B_f^{(II)th}$ .

	Comparison for $Z = 89-98$		
	$B_{f}^{(II)th}$ vs EXP1 [47]	$B_{f}^{(II)th}$ vs EXP2 [48]	
	71	48	
	0.82	0.70	
ns	0.92	0.82	





# Possibilities of direct production of superheavy nuclei with Z=112-118 in different evaporation channels



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#### ABSTRACT

The production cross sections of heaviest isotopes of superheavy nuclei with charge numbers 112–118 are predicted in the *xn*-, *pxn*-, and  $\alpha xn$ -evaporation channels of the <sup>48</sup>Ca-induced complete fusion reactions for future experiments. The estimates of synthesis capabilities are based on a uniform and consistent set of input nuclear data. Nuclear masses, deformations, shell corrections, fission barriers and decay energies are calculated within the macroscopic-microscopic approach for even-even, odd-*Z* and odd-*N* nuclei. For odd systems the blocking procedure is used. To find saddle points, the Imaginary Water Flow technique is used and non-axiallity is taken into account. As shown, our calculations, based on a new set of mass and barriers, agree very well with the experimentally known cross-sections, especially in the 3*n*-evaporation channel. The dependencies of these predictions on the mass/fission barriers tables, the ratio  $a_f/a$ , and fusion models are discussed. A way is shown to produce directly unknown superheavy isotopes in the 1*n*- or 2*n*-evaporation channels. The synthesis of new superheavy isotopes unattainable in reactions with emission of neutrons is proposed in the promising channels with emission of protons ( $\sigma_{pxn} \simeq 10 - 200$  fb) and alphas ( $\sigma_{\alphaxn} \simeq 50 - 500$  fb).

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The measured (symbols) and calculated (lines) excitation functions for xn-evaporation channels (x = 1-5) of the indicated complete fusion reactions.

The NEW mass table is used The black triangles at energy axis indicate the excitation energy at bombarding energy corresponding to the Coulomb barrier for the sphere-side orientation.

The blue diamonds, green squares, red circles, and gray pentagons represent the experimental data with error bars for 2n-, 3n-, 4n-, and 5n-evaporation channels, respectively.

The vertical lines with arrow indicate the upper limits of evaporation residue cross sections









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Topical Review

Topical Review

## Future of nuclear fission theory

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Figure 1. Schematic illustration of the features most relevant to the fission phenomenon.

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#### Instanton-motivated study of spontaneous fission of odd-A nuclei

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Using the idea of the instanton approach to quantum tunneling we try to obtain a method of calculating spontaneous fission rates for nuclei with an odd number of neutrons or protons. This problem has its origin in the failure of the adiabatic cranking approximation which serves as the basis in calculations of fission probabilities. Self-consistent instanton equations, with and without pairing, are reviewed and then simplified to non-self-consistent versions with the phenomenological single-particle potential and seniority pairing interaction. Solutions of instanton-like equations without the pairing and actions they produce are studied for the Woods-Saxon potential along realistic fission trajectories. Actions for unpaired particles are combined with cranking actions for even-even cores and fission hindrance for odd-A nuclei is studied in such a hybrid model. With the mass parameters for neighboring odd-A and even-even nuclei assumed equal, the model shows that freezing the  $K^{\pi}$  configuration leads to a large overestimate of the fission hindrance factors. Actions with adiabatic configurations mostly show not enough hindrance; instanton-like actions for blocked nucleons correct this, but not sufficiently.

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#### I. INTRODUCTION

Nuclear fission is thought to be a collective process, classically envisioned in analogy to the fragmentation of a liquid drop. In reactions induced by neutrons and light or heavy ions, fission is one of many possible deexcitation channels of a formed compound nucleus. On the other hand, spontaneous fission is a decay of the nuclear ground state (g.s.), which exhibits its meta-stability and involves quantum tunneling through a potential barrier. In a theoretical approach, the fission barrier follows from a model of the shape-dependent nuclear energy. In practical terms, it is calculated either from a self-consistent mean-field functional or a microscopicmacroscopic model, as a landscape formed by the lowest energies  $E(\mathbf{q})$  at fixed values of a few arbitrarily chosen coordinates  $\mathbf{q} = (q_1, \ldots, q_i, \ldots)$  (for simplicity assumed dimensionless) describing the nuclear shape. The obscure part of the current approach relates to (a) the likely insufficiency of included coordinates and (b) a description of tunneling dynamics, essentially shaped after the Gamow method, but without a clear understanding of mass parameters and conjugate momenta entering the formula for decay rate.



$$\begin{split} \dot{q} &= \frac{\mu}{\partial \tau} + \dot{q} \sum_{\nu} \left\langle \psi_{\mu}(q(\tau)) \middle| \frac{\tau}{\partial q} (q(\tau)) \right\rangle^{C_{\nu i}} \qquad p_{\mu i}(\tau) = C_{\mu i}^{*}(-\tau) C_{\mu i}(\tau) \\ &= [\zeta_{i} - \epsilon_{\mu}(q(\tau))] C_{\mu i}. \\ S &= \int_{-T/2}^{T/2} \sum_{i \text{ occ}} \sum_{\mu=1}^{\mathcal{N}} [\zeta_{i} - \epsilon_{\mu}(q(\tau))] p_{\mu i}(\tau) d\tau. \end{split}$$

FIG. 7. Pseudo-occupations of the adiabatic states for instantonlike iTDSE solutions for  $\phi_3$  (top), and for  $\phi_5$  (bottom). Colors and line styles correspond to the levels of Fig. 6.

0 τ [10<sup>-21</sup> s] 5

10

15

0

-1

-15

-10

-5







FIG. 9. (left) The total action integrand in units of  $10^{21}$  s<sup>-1</sup>—the sum of individual contributions—for six (in black, the lower one at  $\tau = 0$ ) and seven (in red) neutrons, taken from Ref. [31]. (right) Contributions to the integrand of action from individual s.p. solutions.







FIG. 13. Logarithms of fission hindrance factors,  $\log_{10} HF$ , defined by Eq. (49): experimental (blue circles) vs calculated with (red squares) and without (green triangles) the odd-particle instanton contribution for nuclei specified at the bottom of the panel. The arrow for <sup>257</sup>Rf(m) signifies that only the lower bound for *HF* is experimentally known. See text for further details.

NARODOW CENTRUM BADAŃ JĄDROWY ŚWIERK Here, we use *HF* calculated as

$$HF = \frac{T_{\rm sf}^{\,o}}{T_{\rm sf}^{\,e}},\tag{49}$$

where  $T_{sf}^{o}$  and  $T_{sf}^{e}$  are fission half-lives of an odd-A nucleus and its A - 1 e-e neighbor.

## Dziękuję za uwagę



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